Problem Set 5.

Ben Polak Econ 159a/MGT522a

Three questions due October 24, 2007
(N.B. This is after the midterm.)

1. Evolutionary Stability and Weak Domination (Osborne). Suppose that the pure strategy s^* is evolutionarily stable. Is it possible that there is some other pure strategy that weakly dominates s^*? Is it possible that there is some other pure strategy that is **not** weakly dominated by s^*? Briefly explain your answer.

2. Evolutionary Stability and Pareto Ranked Equilibria (adapted from Osborne). Consider the following symmetric two-player game. Each player can 'demand' an amount 1, 2 or 3. If both players demand the same amount then they each get that amount. If they demand different amounts then the player who demands less gets the amount demanded by the player who demanded more, and the player who demands more gets 1/4 of her demand. That is,

$$u_1(s_1, s_2) = \begin{cases} s_2 & \text{if} \quad s_1 < s_2 \\ s_1 & \text{if} \quad s_1 = s_2 \\ \frac{s_1}{4} & \text{if} \quad s_1 > s_2 \end{cases}.$$

(a) Draw the normal form matrix for this game and find all the symmetric pure-strategy Nash Equilibria. Are they Pareto ranked?

(b) Which pure strategies are evolutionary stable against pure-strategy mutant invasions?

(c) How would your answer to part (b) change if a possible demand of 0 were added to the other three strategies (keeping the same payoff rule)?

(d) How would your answer to part (b) change if the payoff $u_1(s_1, s_2)$ when $s_1 > s_2$ was changed to $\frac{s_1}{3}$ (otherwise keeping the payoff rule the same)?

3. Clever Mutants. Consider the symmetric two-player game:

		2: a	b
1	a	$3, 3$	$0, 0$
	b	$0, 0$	$1, 1$

(a) Find all the symmetric Nash equilibria, including any mixed-strategy equilibria.

(b) Find all the evolutionarily stable strategies, including any mixed-strategy (i.e., polymorphic) ESS. Explain your answer.

(c) Now, suppose that mutants have a 'secret handshake'. That is, suppose that mutants can recognize other mutants and play different pure strategies against normal and mutant opponents. For example, a mutant could play b against another mutant but play a against a non-mutant. Argue informally there can no longer be an ESS in which only b is played.