Problem Set 2 .

Ben Polak Econ 159a/MGT522a

Four questions, due September 26, 2007.

1. Penalty Shots Revisited. Player 1 has to take a soccer penalty shot to decide the game. She can shoot Left, Middle, or Right. Player 2 is the goalie. He can dive to the $l$eft, $m$iddle, or $r$ight. Actions are chosen simultaneously. The payoffs (which here are the probabilities in tenths of winning) are as follows.

		2		
		l	m	r
	L	$4,6$	$7,3$	$9,1$
1	M	$6,4$	$3,7$	$6,4$
	R	$9,1$	$7,3$	$4,6$

(a) For each player, is any strategy dominated by another (pure) strategy?

(b) For what beliefs about player 1's strategy is m a best response for player 2? For what beliefs about player 2's strategy is M a best response for player 1? [Hint: you do not need to draw a 3-dimensional picture!].

(c) Suppose player 2 "puts himself in player 1's shoes" and assumes that player 1, what ever is her belief, will always choose a best-response to that belief. Should player 2 ever choose m?

(d) Show that this game does not have a (pure-strategy) Nash Equilibrium?

2. Partnerships Revisited. (Adapted from Watson.) Recall the partnership game we discussed in class. Two law partners jointly own a firm and share equally in its revenues. Each law partner individually decides how much effort to put into the firm. The firm's revenue is given by $4(s_1 + s_2 + bs_1s_2)$ where s_1 and s_2 are the efforts of the lawyers 1 and 2 respectively. The parameter $b > 0$ reflects the synergies between their efforts: the more one lawyer works, the more productive is the other. Assume that $0 \leq b \leq 1/4$, and that each effort level $S_i = [0, 4]$. The payoff for partners 1 and 2 are:

$$
\begin{aligned}
u_1(s_1, s_2) &= \frac{1}{2}[4(s_1 + s_2 + bs_1s_2)] - s_1^2 \\
u_2(s_1, s_2) &= \frac{1}{2}[4(s_1 + s_2 + bs_1s_2)] - s_2^2
\end{aligned}
$$

respectively, where the s_i^2 terms reflect the cost of effort. (Notice that the cost of providing another unit of effort is increasing in the amount of effort already provided). Assume the firm has no other costs. In class, we showed that the only *rationalizable* strategies (i.e., those not deleted by the process of iteratively deleting strategies that are never a best response) were $s_1^* = s_2^* = \frac{1}{1-b}$.

(a) Suppose that the partners both agree to work the same amount as each other, and that they write a contract specifying that amount. What common amount of effort s^{**} should they agree each to supply to the firm if their aim is to maximize revenue net of total effort costs. How does this amount compare to the rationalizable effort levels we found in class. Give a brief intuition for this comparison. [Hint: for the intuition, it may help to consider the special case $b = 0$.]

(b) Suppose now that the contract is only binding on partner 2. That is, partner 2 has to provide the effort level s^{**} you found from part (a), but partner 1 is free to choose any effort level between 0 and 4. What effort level will partner 1 choose? How does this amount compare to s_1^* and to s_1^{**}. Give a brief intuition for you answer.

(c) Return to the basic game we discussed in class, but now assume that $b = -1/2$; that is, the partners' efforts have negative synergies. Solve for the best-response functions in this case, and draw the best-response diagram. Find the set of rationalizable strategies. Again, compare these effort levels with those that the partners would choose if they could contract to provide the same amount as each other. [Hint: you do not need to re-do all the work of part (a).]

3. Nash Equilibria and Iterative Deletion (Gibbons) Consider the following game.

	L	C	R
T	$2,0$	$1,1$	$4,2$
M	$3,4$	$1,2$	$2,3$
B	$1,3$	$0,2$	$3,0$

(a) What strategies survive iterative deletion of strictly dominated strategies?

(b) Find the (pure strategy) Nash equilibria of this game.

(c) Argue as carefully but as concisely as you can that, in general (not just in this game), strategies that form part of a Nash equilibrium are never eliminated by iterative deletion of *strictly* dominated strategies?

4. Splitting the Dollar(s) (adapted from Gibbons and Osborne). Players 1 and 2 are bargaining over how to split \$10. Each player i names an amount, s_i, between 0 and 10 for herself. These numbers do not have to be in whole dollar units. The choices are made simultaneously. Each player's payoff is equal to her own money payoff. We will consider this game under two different rules. In both cases, if $s_1 + s_2 \leq 10$ then the players get the amounts that they named (and the remainder, if any, is destroyed).

(a) In the first case, if $s_1 + s_2 > 10$ then both players get zero and the money is destroyed. What are the (pure strategy) Nash Equilibria of this game?

(b) In the second case, if $s_1 + s_2 > 10$ and the amounts named are different, then the person who names the smaller amount gets that amount and the other person gets the remaining money. If $s_1 + s_2 > 10$ and $s_1 = s_2$, then both players get \$5. What are the (pure strategy) Nash Equilibria of this game?

(c) Now suppose these two games are played with the extra rule that the named amounts have to be in whole dollar units. Does this change the (pure strategy) Nash Equilibria in either case?