Module 27: Poynting Vector and Energy Flow

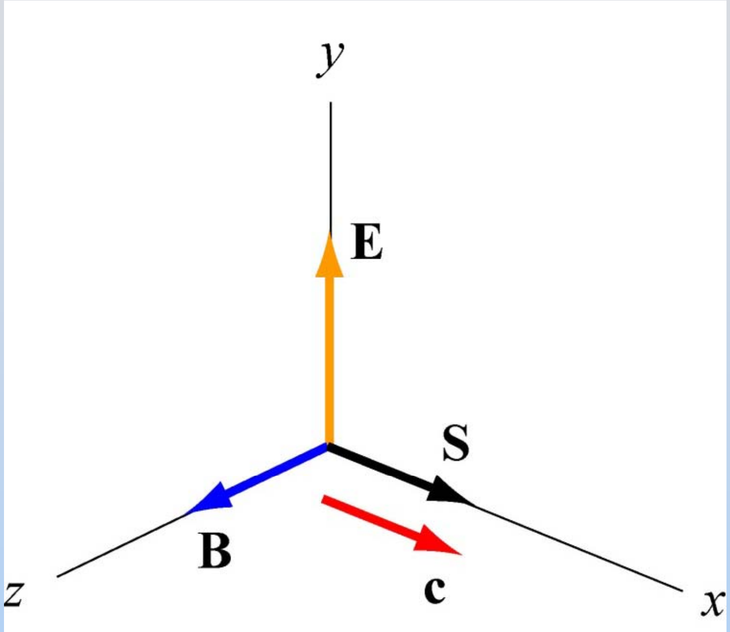
Module 27: Outline

Poynting Vector and Energy Flow

Examples

Energy Flow

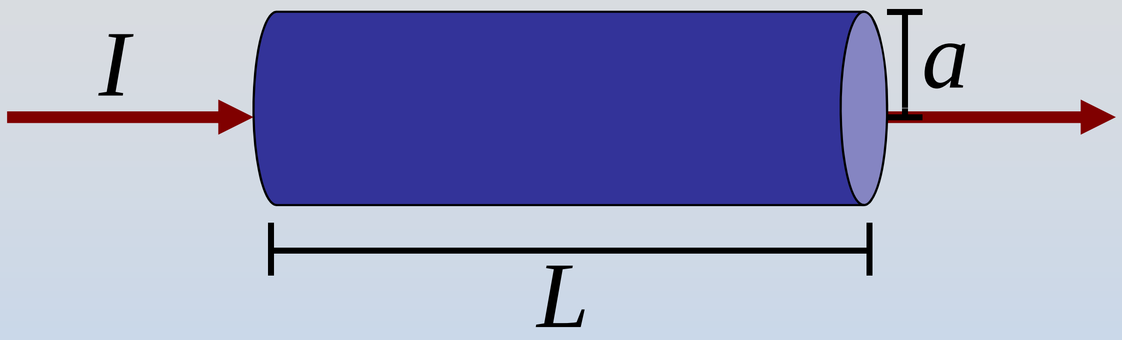
Poynting Vector



Power flow per unit area:

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$: Poynting vector

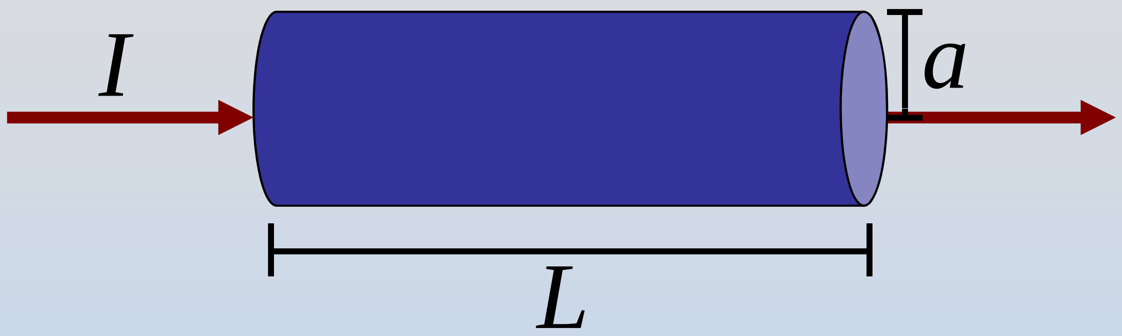
Problem: Resistor Power



Consider the above cylindrical resistor, with current I and voltage drop ΔV. Calculate the power in terms of the electric and magnetic fields at the surface of the resistor.

There is a geometric factor. What is it?

In Class Solution: Resistor Power



$$\Delta V = EL \qquad B = \frac{\mu_0 I}{2\pi a} \Rightarrow I = \frac{2\pi a B}{\mu_0}$$

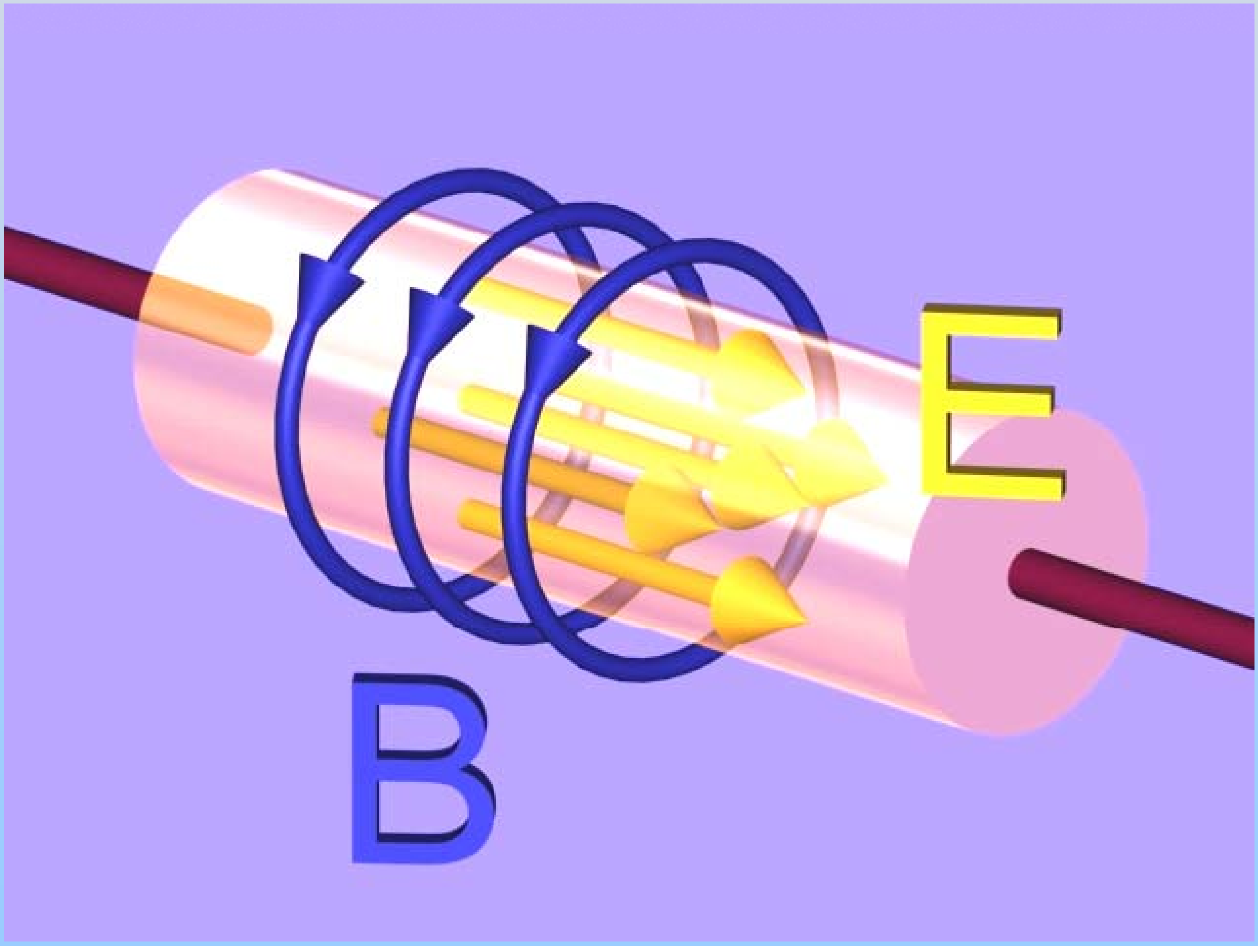
$$P = \Delta V \cdot I = EL \cdot \frac{2\pi a B}{\mu_0} = 2\pi a L \cdot \frac{EB}{\mu_0}$$

Surface area

Energy Flow: Resistor

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

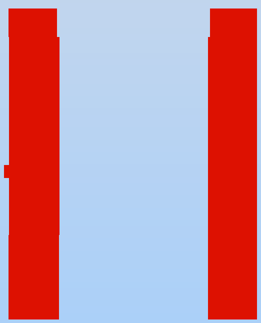
On surface of resistor is INWARD



Power & Energy in Circuit Elements

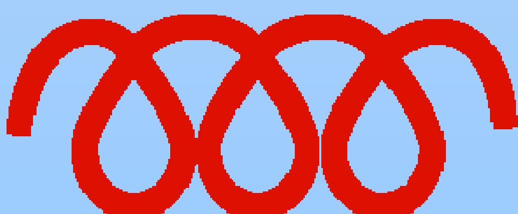
$$P = \iint_{Surface} \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}}$$

Dissipates Power



$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

Store Energy

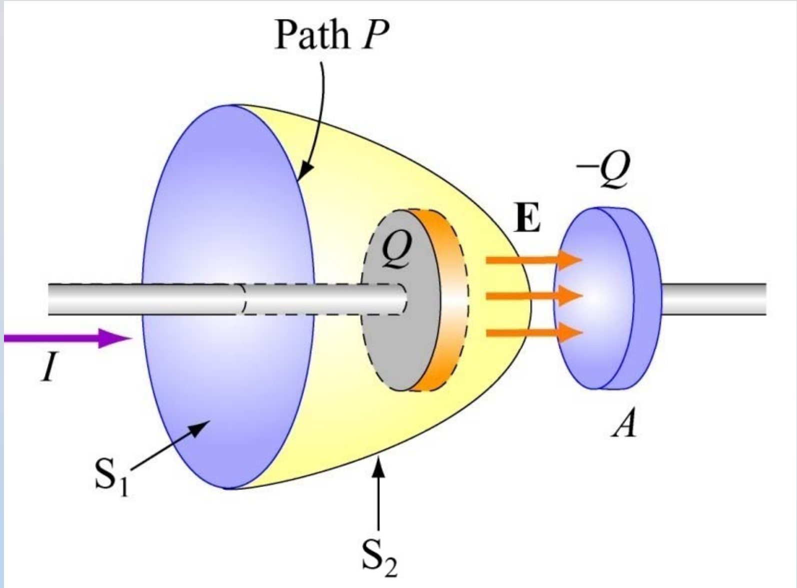


$$u_B = \frac{1}{2\mu_0} B^2$$

POWER When (dis)charging

Don't Forget Displacement Current

Displacement Current



$$E = \frac{Q}{\varepsilon_0 A} \Rightarrow Q = \varepsilon_0 EA = \varepsilon_0 \Phi_E$$

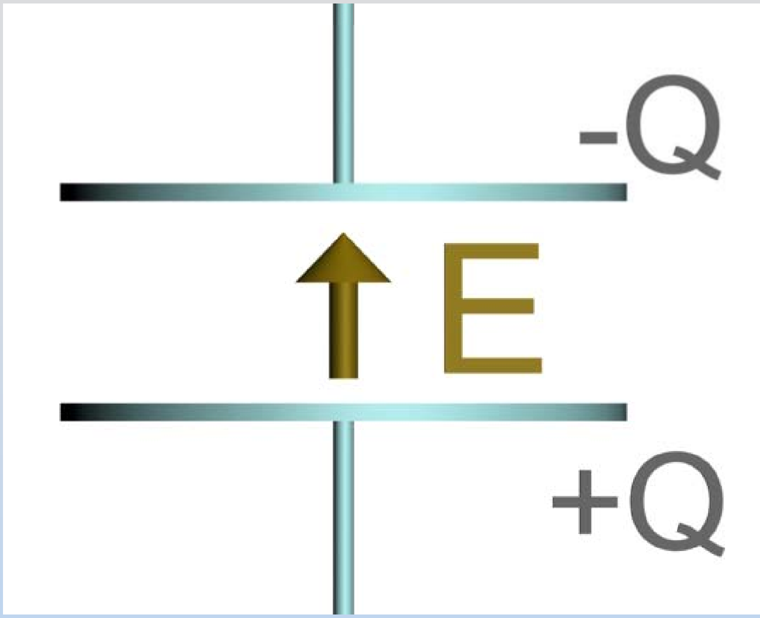
$$\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_d$$

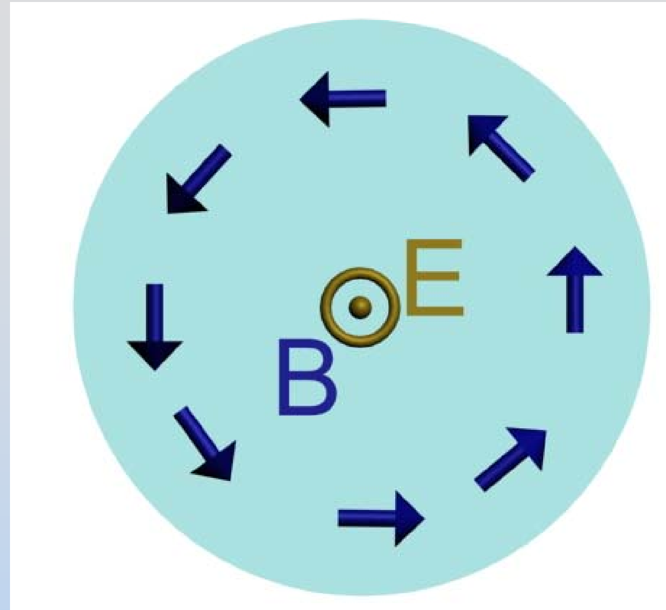
So we have to modify Ampere's Law:

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 (I_{encl} + I_d)$$

Concept Question Questions: Poynting Vector

Concept Question: Capacitor

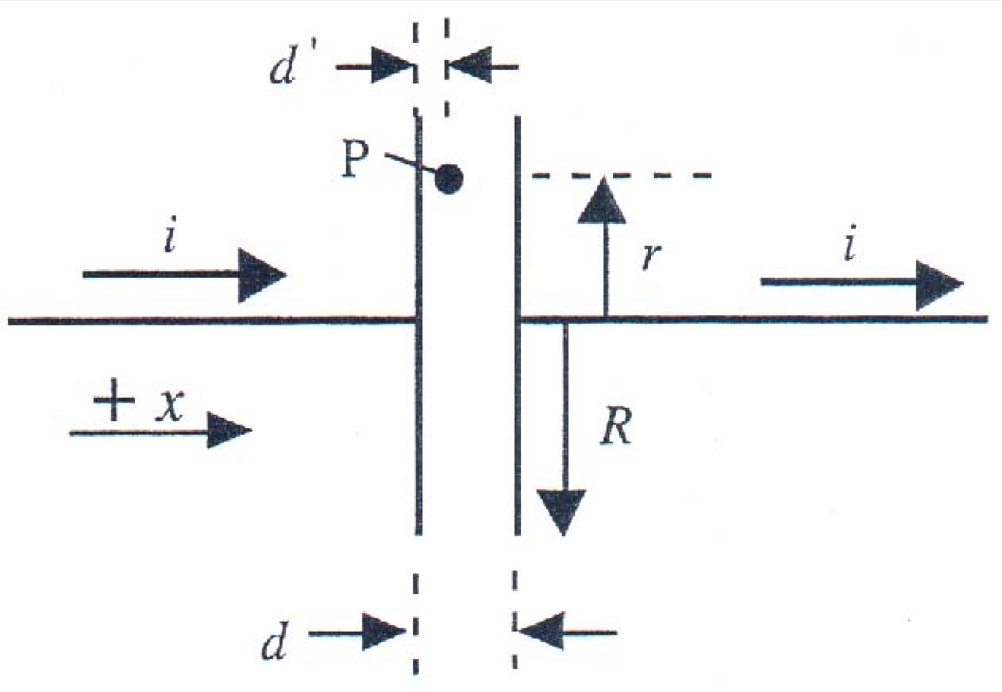




The figures above show a side and top view of a capacitor with charge *Q* and electric and magnetic fields E and B at time *t*. At this time the charge *Q* is:

1. Increasing in time
2. Constant in time.
3. Decreasing in time.
4. I don't know

Problem: Capacitor



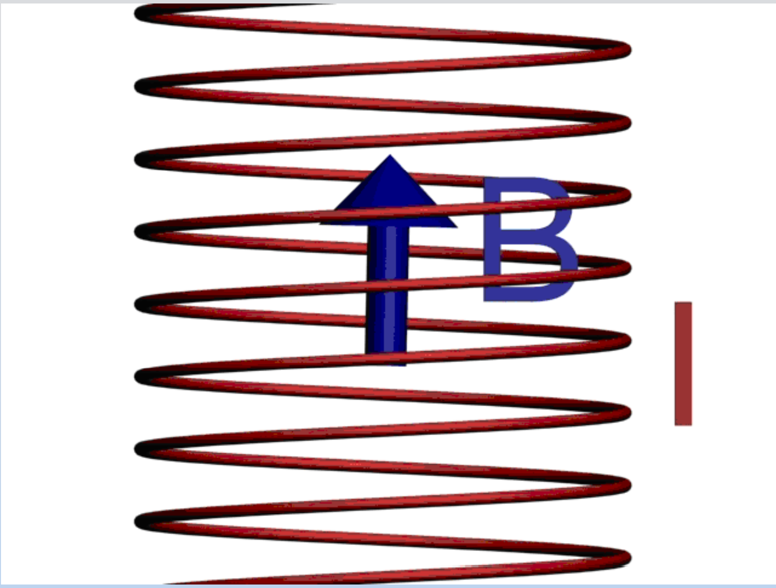
A circular capacitor of spacing *d* and radius *R* is in a circuit carrying the steady current *i* shown.

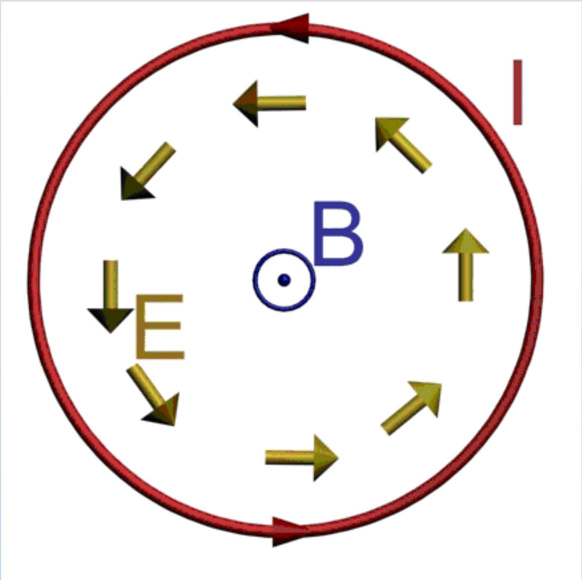
At time t=0 it is uncharged

1. Find the electric field **E**(t) at P vs. time t (mag. & dir.)
2. Find the magnetic field **B**(t) at P
3. Find the Poynting vector **S**(t) at P
4. What is the total power flux into/out of the capacitor?
5. Does this make sense? How? (Hint: What's U?)

Another look at Inductance

Concept Question: Inductor

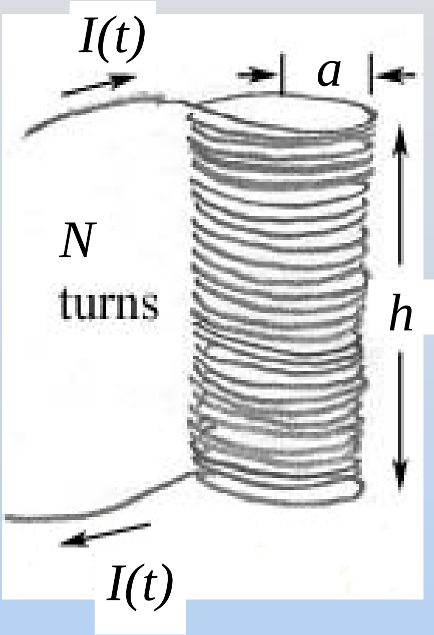




The figures above show a side and top view of a solenoid carrying current I with electric and magnetic fields E and B at time t. In the solenoid, the current I is:

1. Increasing in time
2. Constant in time.
3. Decreasing in time.
4. I don't know

Problem: Inductor



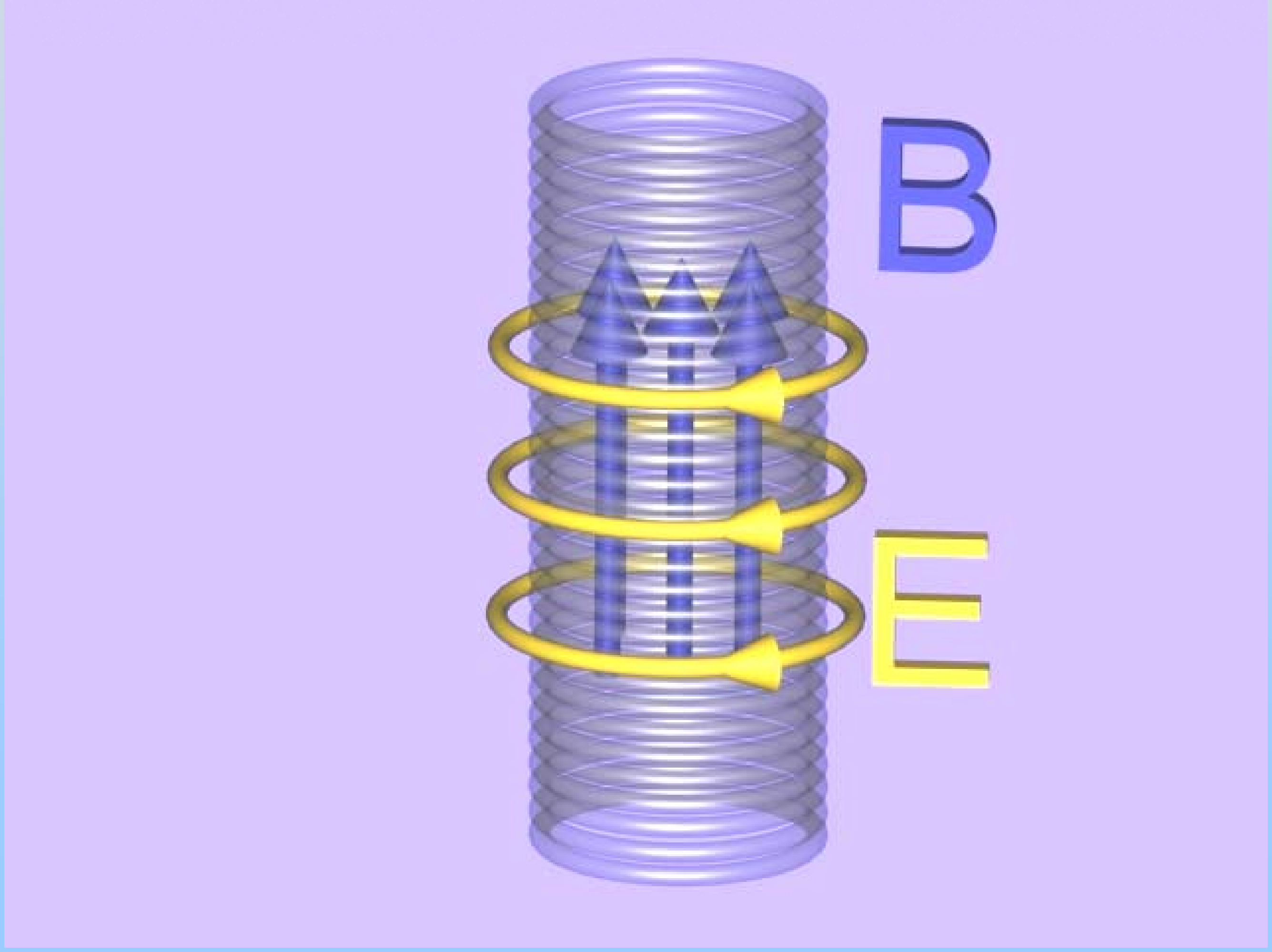
A solenoid of radius a and length h has an increasing current $I(t)$ as pictured.
Consider a point P at radius r ($r < a$).

1. Find the magnetic field **B**(t) at P vs. time t
2. Find the electric field **E**(t) at P
3. Find the Poynting vector **S**(t) at P
4. What is the total power flux into/out of the inductor?
5. Does this make sense? How? (Hint: What's U?)

Energy Flow: Inductor

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

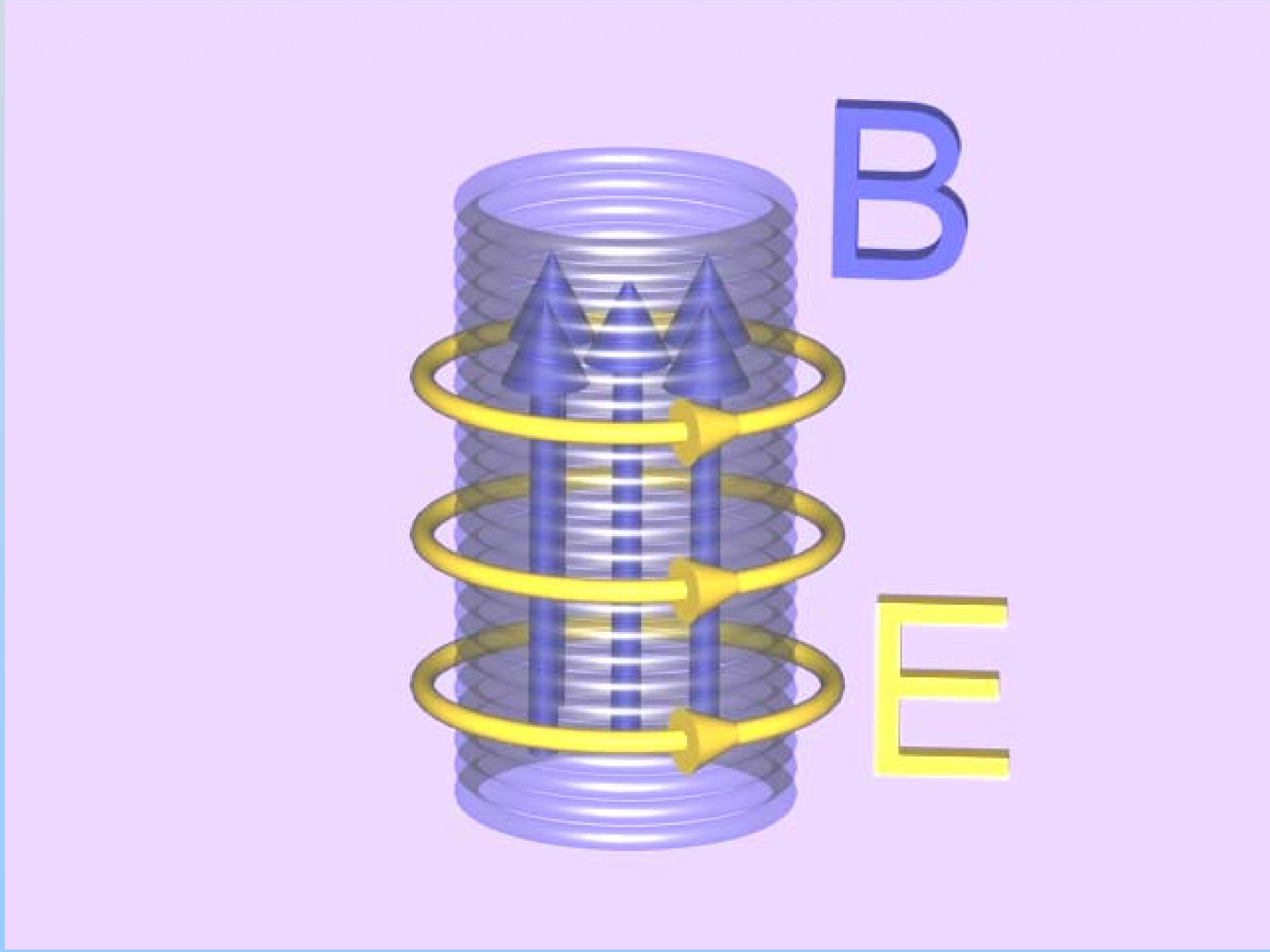
On surface of inductor with increasing current is INWARD



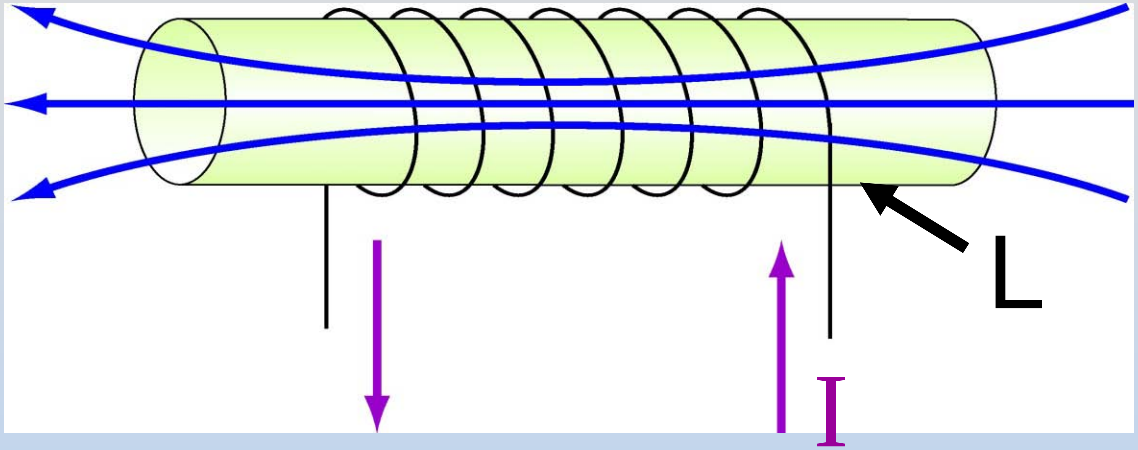
Energy Flow: Inductor

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

On surface of inductor with decreasing current is OUTWARD



Faraday & Inductors



$$LI = \Phi_{Self}$$

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$