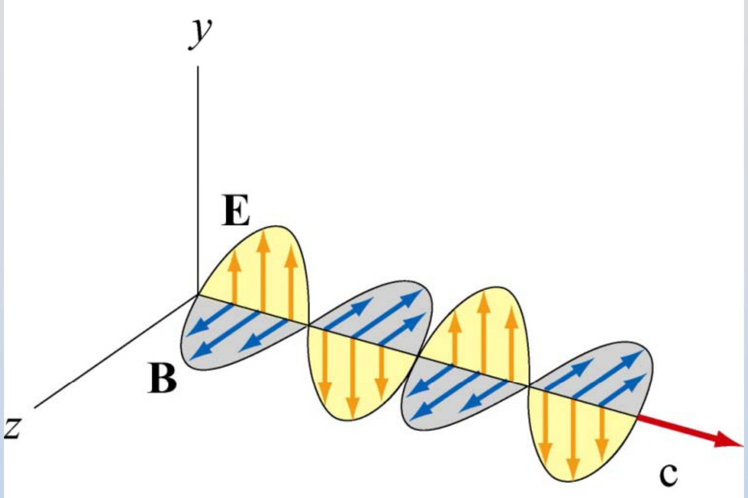
Module 29:
Energy and Momentum in EM Waves

Module 29: Outline

Energy and Momentum in EM Waves

Summary:
Traveling Electromagnetic Waves

Properties of EM Waves



Travel (through vacuum) with speed of light

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$

At every point in the wave and any instant of time, E and B are in phase with one another, with

$$\frac{E}{B} = \frac{E_0}{B_0} = c$$

E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

$$\text{Direction of propagation} = \text{Direction of } \vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

Traveling E & B Waves

Wavelength: λ

Frequency : f

$$\boxed{\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)}$$

Wave Number: $k = \dfrac{2\pi}{\lambda}$

Angular Freq.: $\omega = 2\pi f$

Period: $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$

Speed: $v = \dfrac{\omega}{k} = \lambda f$

Direction: $+\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$

$$\frac{E}{B} = \frac{E_0}{B_0} = v$$

In vacuum...

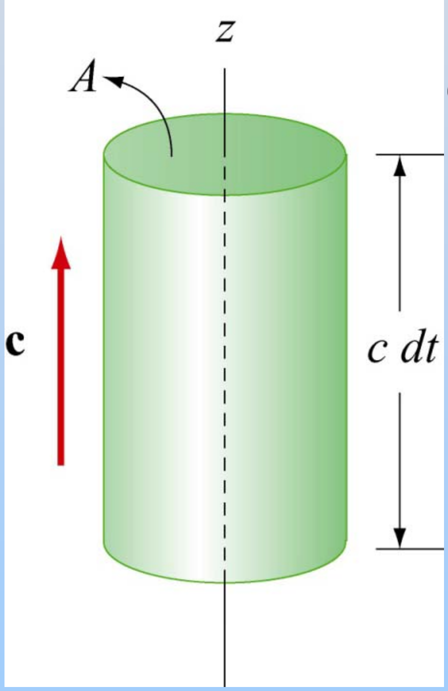
$$= c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \, \frac{m}{s}$$

Energy & the Poynting Vector

Energy in EM Waves

Energy densities: $u_E = \frac{1}{2}\varepsilon_0 E^2,\ u_B = \frac{1}{2\mu_0} B^2$

Consider cylinder:



$$dU = (u_E + u_B)A\,dz = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)Acdt$$

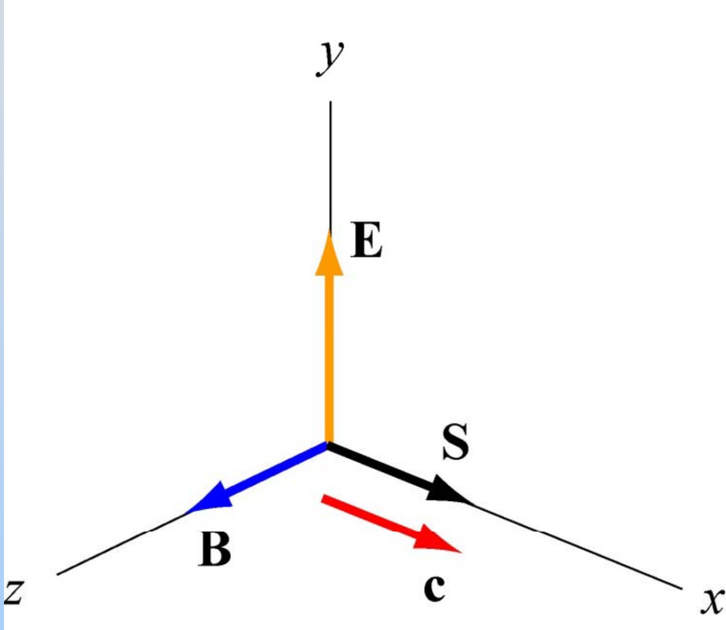
What is rate of energy flow per unit area?

$$S = \frac{1}{A}\frac{dU}{dt} = \frac{c}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right) = \frac{c}{2}\left(\varepsilon_0 cEB + \frac{EB}{c\mu_0}\right)$$

$$= \frac{EB}{2\mu_0}\left(\varepsilon_0\mu_0 c^2 + 1\right) = \frac{EB}{\mu_0}$$

Poynting Vector and Intensity

Direction of energy flow = direction of wave propagation



$$\boxed{\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0} \text{ : Poynting vector}}$$

units: Joules per square meter per sec

Intensity *I*:

$$\boxed{I \equiv \langle S \rangle = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{cB_0^2}{2\mu_0}}$$

Momentum & Radiation Pressure

EM waves transport energy: $\vec{\mathbf{S}} = \dfrac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$

They also transport momentum: $p - \dfrac{U}{c}$

And exert a pressure: $P = \dfrac{F}{A} = \dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{1}{cA}\dfrac{dU}{dt} = \dfrac{S}{c}$

This is only for hitting an absorbing surface. For hitting a perfectly reflecting surface the values are doubled:

Momentum transfer: $p = \dfrac{2U}{c}$; Radiation pressure: $P = \dfrac{2S}{c}$

Problem: Catchin' Rays

As you lie on a beach in the bright midday sun, approximately what force does the light exert on you?

The sun:

Total energy output of ~ 4×10^{26} Watts

Distance from Earth 1 AU ~ 150×10^{6} km

Speed of light $c = 3 \times 10^{8}$ m/s

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