Module 03: Electric Fields and Discrete Charge Distributions

Module 03: Outline

Review:

Electric Fields

Charge

Dipoles

Last Time: Gravitational & Electric Fields

Gravitational & Electric Fields

SOURCE: Mass M_s Charge q_s (±)

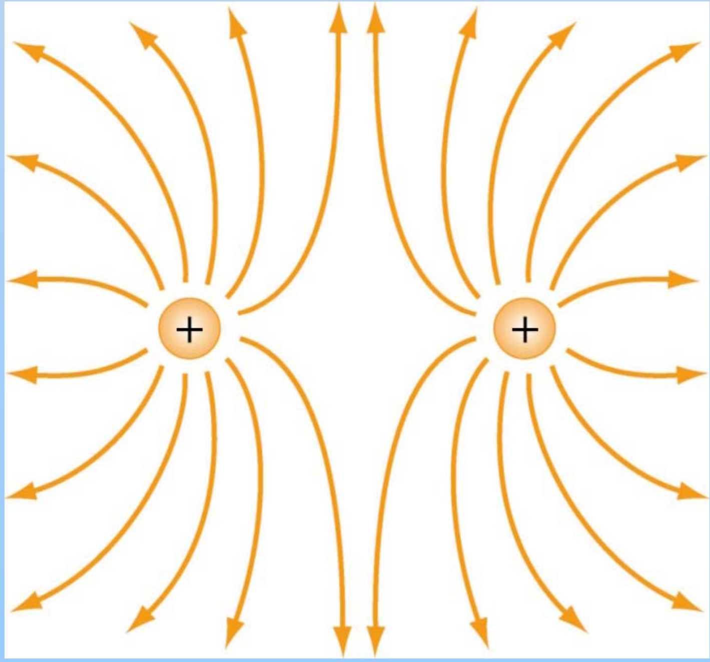
CREATE: $\vec{\mathbf{g}} = -G\frac{M_s}{r^2}\hat{\mathbf{r}}$ $\vec{\mathbf{E}} = k_e\frac{q_s}{r^2}\hat{\mathbf{r}}$

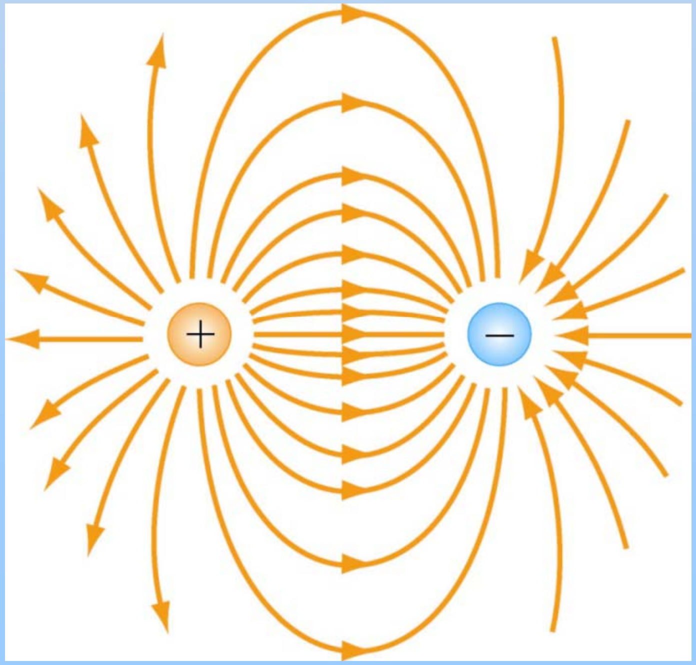
FEEL: $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$ $\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

This is easiest way to envision field, as producing forces!

Electric Field Lines

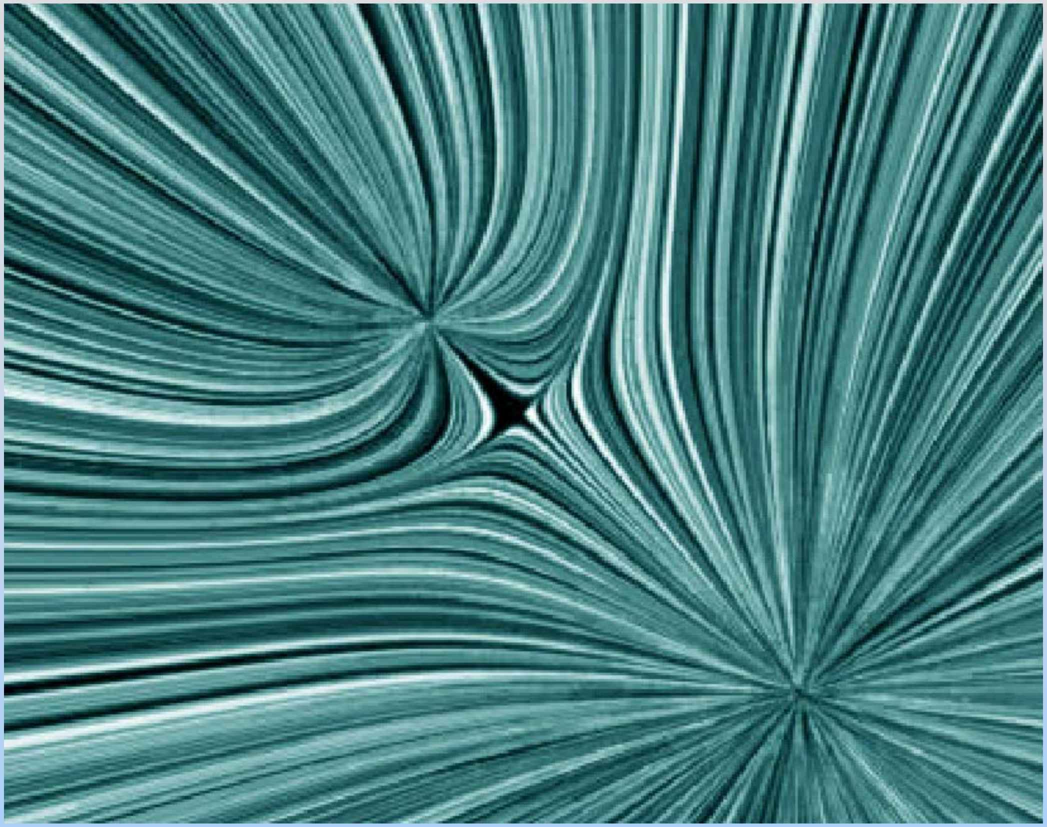
1. Direction of field at any point is tangent to field line at that point
2. Field lines point away from positive charges and terminate on negative charges
3. Field lines never cross each other





Concept Questions on Electric Field

Concept Question: Force



The picture shows the field lines around two charges.

The force between the two charges is:

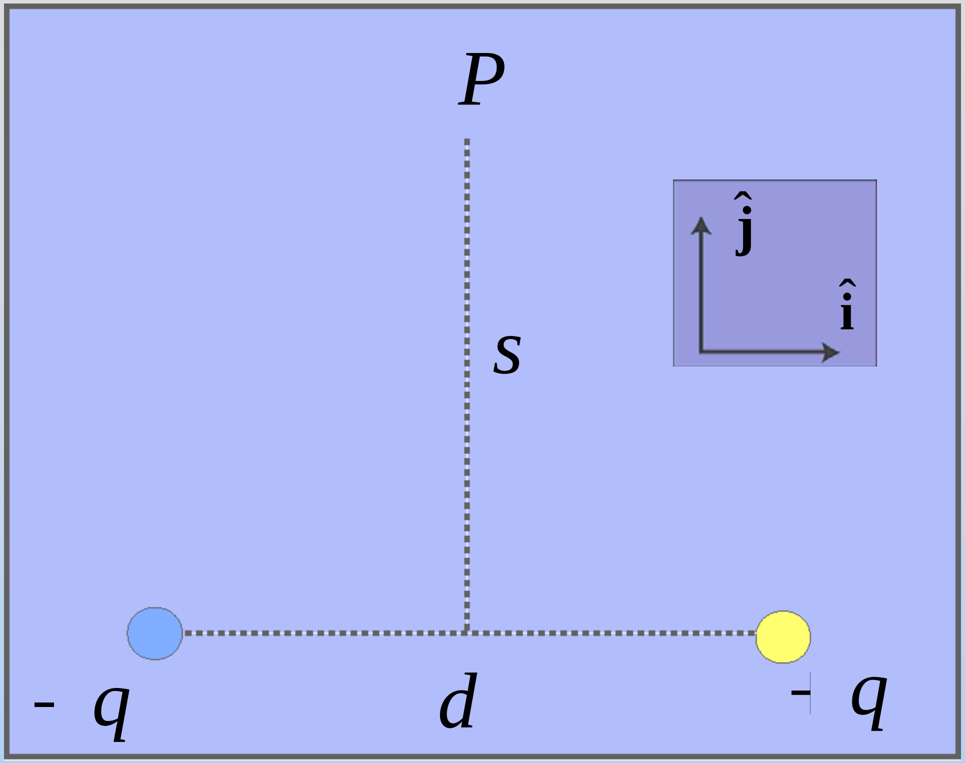
1. Attractive
2. Repulsive
3. Can’t tell without more information
4. I don’t know

Concept Question: Field Lines

Electric field lines show:

1. Directions of forces that exist in space at all times.
2. Directions in which positive charges on those lines will accelerate.
3. Paths that charges will follow.
4. More than one of the above.
5. I don’t know.

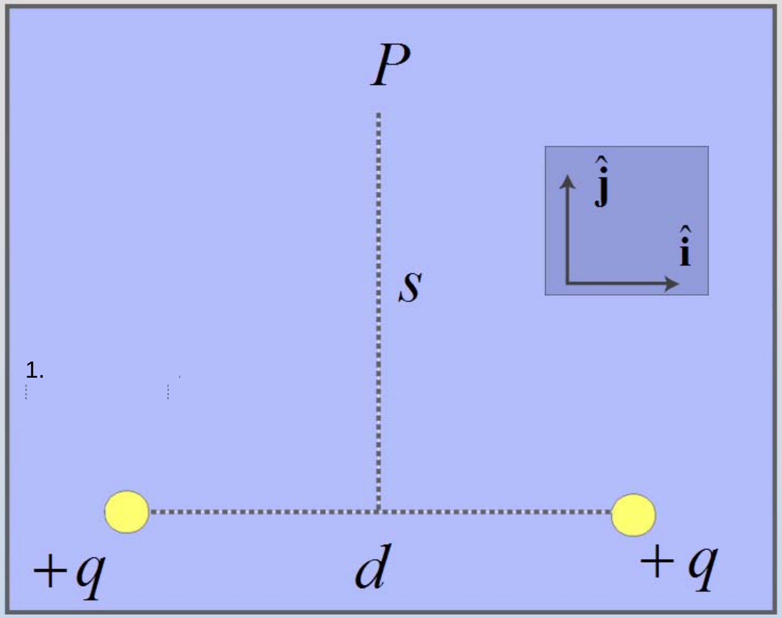
Checkpoint Problem



Consider two point charges of equal magnitude but opposite signs, separated by a distance *d*. Point *P* lies along the perpendicular bisector of the line joining the charges, a distance *s* above that line. What is the E field at *P*?

Two Concept Questions: E Field of Finite # of Point Charges

Concept Question: Equal Charges



Electric field at P is:

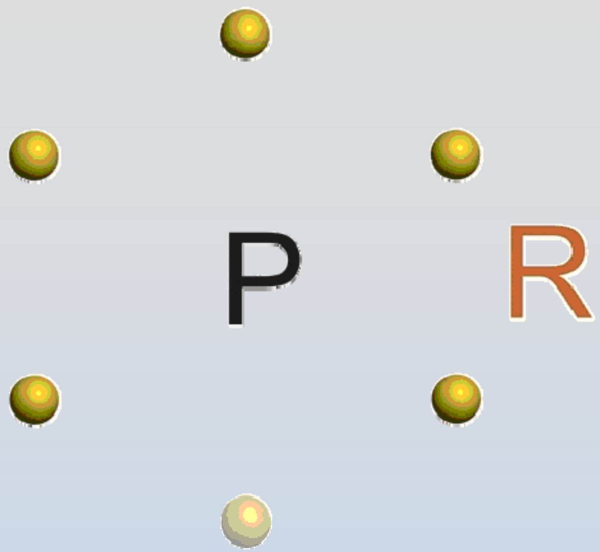
1. $\vec{\mathbf{E}} = \frac{2k_e q s}{\left[s^2 + \frac{d^2}{4}\right]^{3/2}} \hat{\mathbf{j}}$

2. $\vec{\mathbf{E}} = -\frac{2k_e q d}{\left[s^2 + \frac{d^2}{4}\right]^{3/2}} \hat{\mathbf{i}}$

3. $\vec{\mathbf{E}} = \frac{2k_e q d}{\left[s^2 + \frac{d^2}{4}\right]^{3/2}} \hat{\mathbf{j}}$

4. $\vec{\mathbf{E}} = -\frac{2k_e q s}{\left[s^2 + \frac{d^2}{4}\right]^{3/2}} \hat{\mathbf{i}}$

5. I Don't Know



Concept Question: 5 Equal Charges

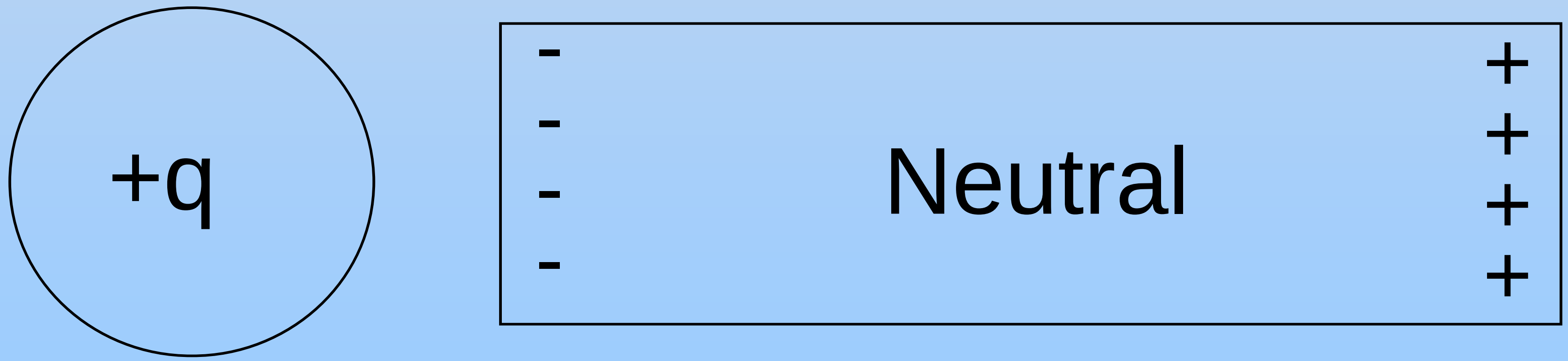
Six equal positive charges *q* sit at the vertices of a regular hexagon with sides of length R. We remove the bottom charge. The electric field at the center of the hexagon (at point P) is:

1. $\vec{\mathbf{E}} = \frac{2kq}{R^2} \; \hat{\mathbf{j}}$
2. $\vec{\mathbf{E}} = -\frac{2kq}{R^2} \; \hat{\mathbf{j}}$
3. $\vec{\mathbf{E}} = \frac{kq}{R^2} \; \hat{\mathbf{j}}$
4. $\vec{\mathbf{E}} = -\frac{kq}{R^2} \; \hat{\mathbf{j}}$
5. $\vec{\mathbf{E}} = 0$
6. I Don't Know

Charging

How Do You Get Charged?

- Friction
- Transfer (touching)
- Induction



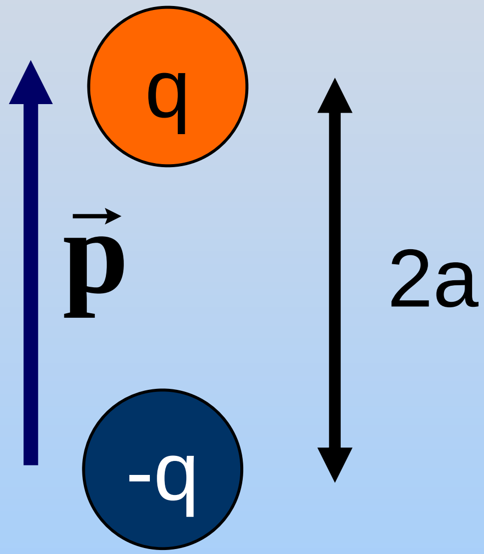
Demonstrations: Instruments for Charging

Electric Dipoles

A Special Charge Distribution

Electric Dipole

Two equal but opposite charges **+*q*** and **–*q***, separated by a distance **2a**



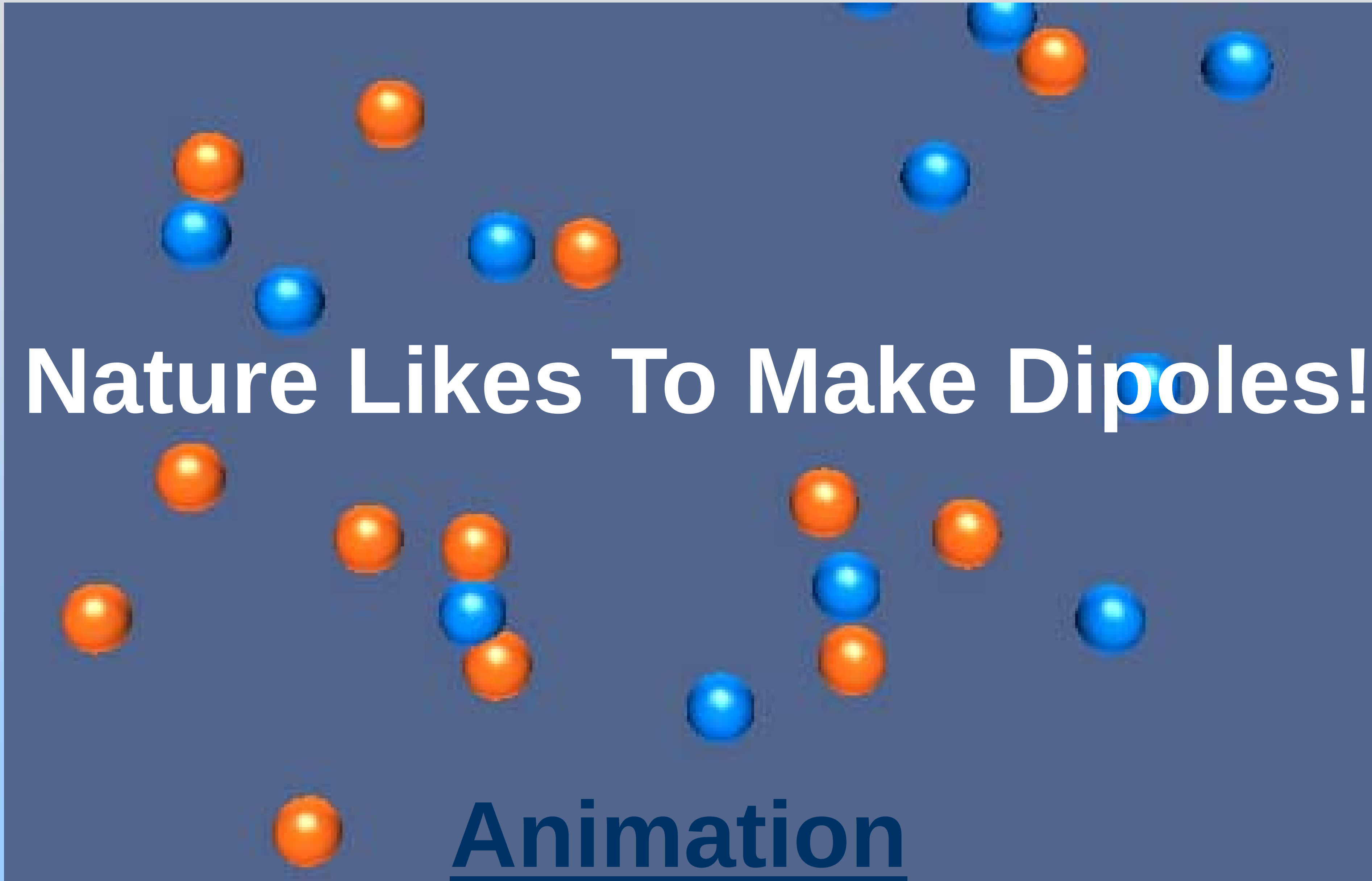
Dipole Moment:

Vector product of charge with displacement

$$\vec{\mathbf{p}} = q \times 2a\hat{\mathbf{j}} = 2qa\hat{\mathbf{j}}$$

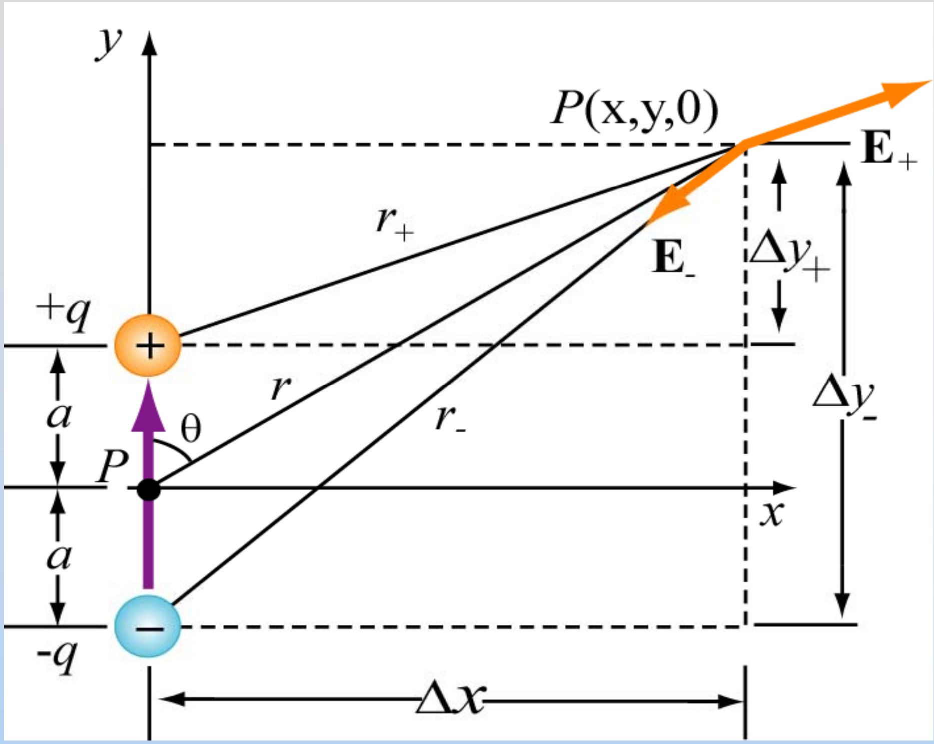
$\vec{\mathbf{p}}$ points from negative to positive charge

Why Dipoles?



Dipoles *make* Fields

Electric Field Created by Dipole



Thou shalt use components!

$$\frac{\hat{\mathbf{r}}}{r^2} = \frac{\vec{\mathbf{r}}}{r^3} = \frac{\Delta x}{r^3}\hat{\mathbf{i}} + \frac{\Delta y}{r^3}\hat{\mathbf{j}}$$

$$E_x = k_e q\left(\frac{\Delta x}{r_+^3} - \frac{\Delta x}{r_-^3}\right) = k_e q\left(\frac{x}{\left[x^2+(y-a)^2\right]^{3/2}} - \frac{x}{\left[x^2+(y+a)^2\right]^{3/2}}\right)$$

$$E_y = k_e q\left(\frac{\Delta y_+}{r_+^3} - \frac{\Delta y_-}{r_-^3}\right) = k_e q\left(\frac{y-a}{\left[x^2+(y-a)^2\right]^{3/2}} - \frac{y+a}{\left[x^2+(y+a)^2\right]^{3/2}}\right)$$

Concept Question Question: Dipole Fall-Off

Concept Question: Dipole Field

As you move to large distances r away from a dipole, the electric field will fall-off as:

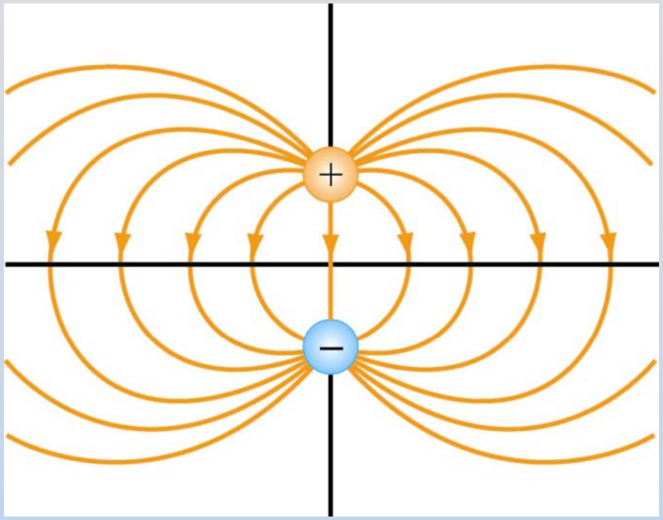
1. $1/r^2$, just like a point charge
2. More rapidly than $1/r^2$
3. More slowly than $1/r^2$
4. I Don’t Know

Concept Question Answer: Dipole Field

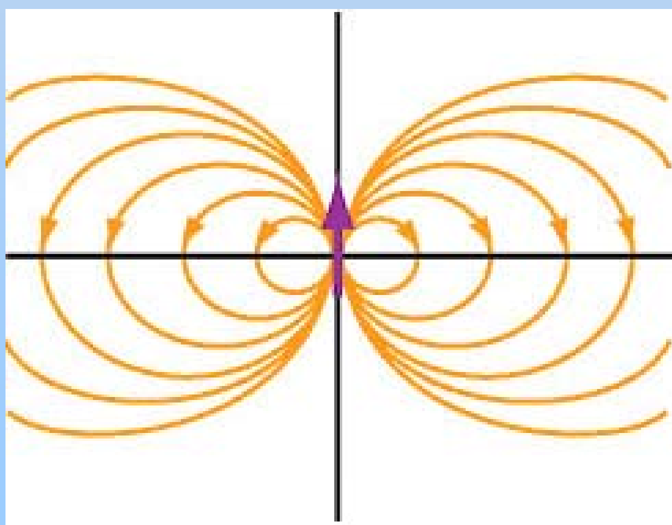
Answer: 2) More rapidly than $1/r^2$

We know this must be a case by thinking about what a dipole looks like from a large distance. To first order, it isn't there (net charge is 0), so the E-Field must decrease faster than if there were a point charge there.

Point Dipole Approximation



Finite Dipole



Point Dipole

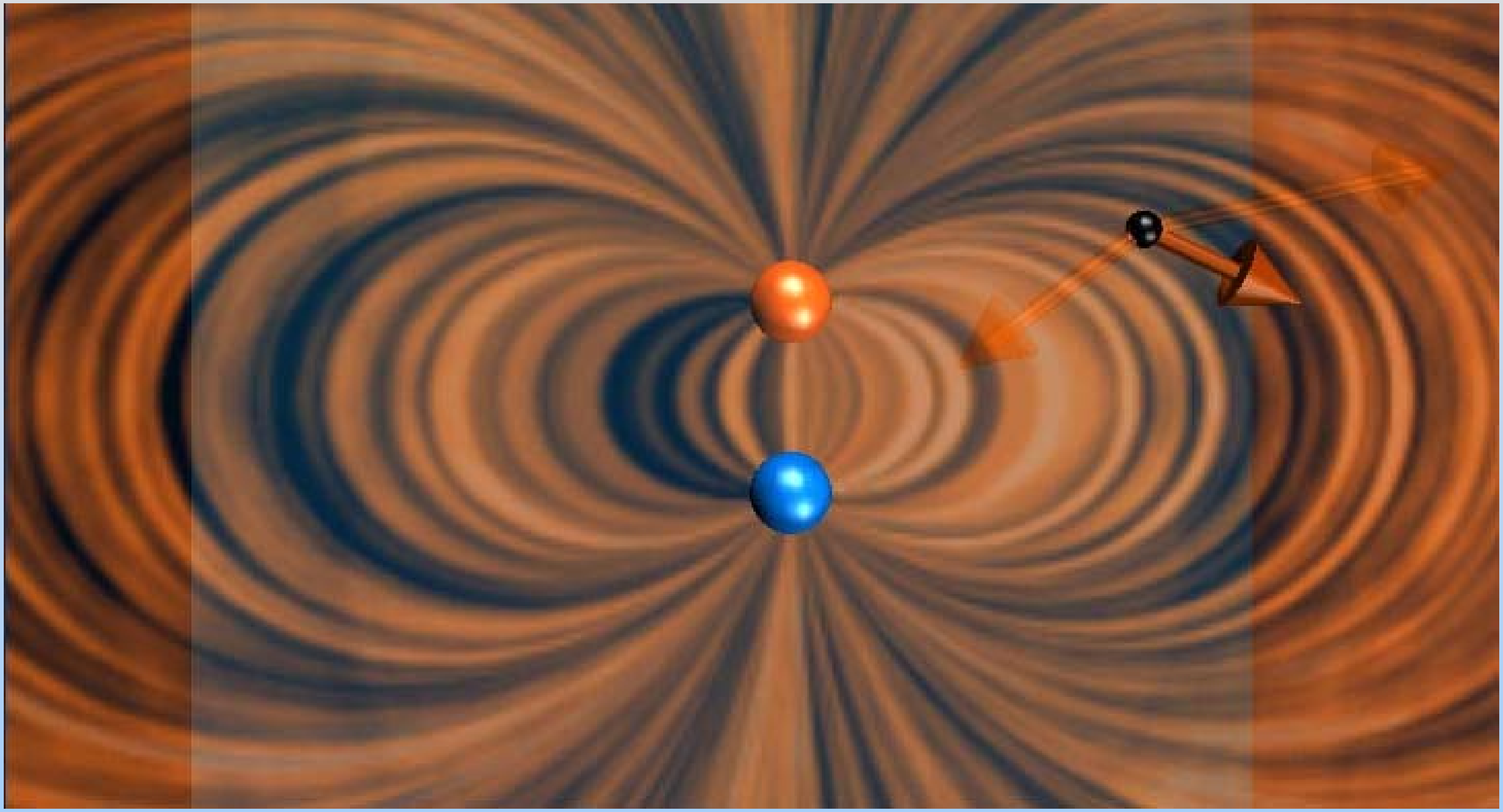
Take the limit $r >> a$

You can show…

$$E_x - \frac{3p}{4\pi\varepsilon_0 r^3}\sin\theta\cos\theta$$

$$E_y - \frac{p}{4\pi\varepsilon_0 r^3}\left(3\cos^2\theta - 1\right)$$

Shockwave for Dipole

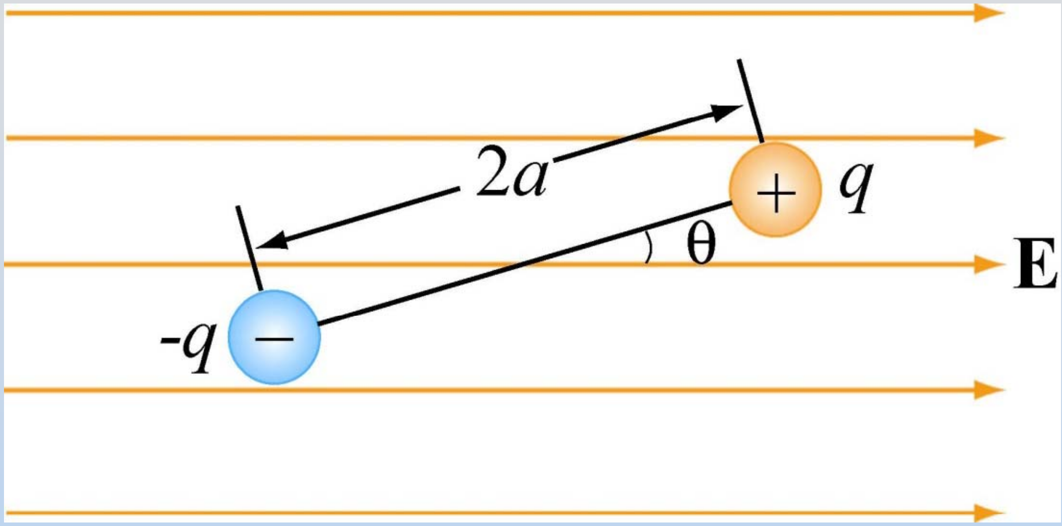


Dipole Visualization

Dipoles *feel* Fields

Demonstration: Dipole in Field

Dipole in Uniform Field



$$\vec{\mathbf{E}} = E\hat{\mathbf{i}}$$

$$\vec{\mathbf{p}} = 2qa(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}})$$

Total Net Force: $\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_{+} + \vec{\mathbf{F}}_{-} = q\vec{\mathbf{E}} + (-q)\vec{\mathbf{E}} = 0$

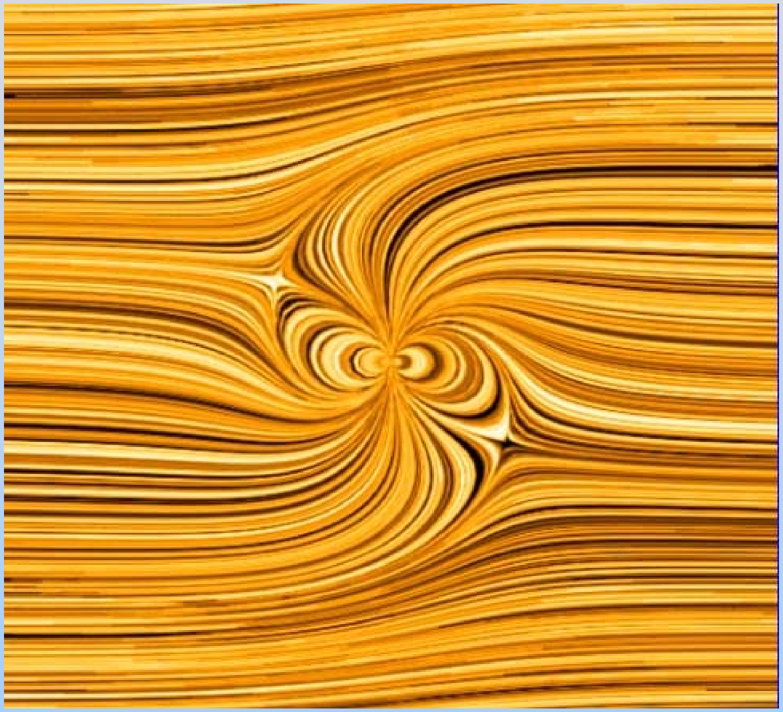
Torque on Dipole: $\boxed{\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \;\; = \vec{\mathbf{p}} \times \vec{\mathbf{E}}}$

$$\tau = rF_{\perp}\sin(\theta) = \left(2a\right)\left(qE\right)\sin(\theta) = pE\sin(\theta)$$

$\vec{\mathbf{p}}$ tends to align with the electric field

Torque on Dipole

Total Field (dipole + background) shows torque:

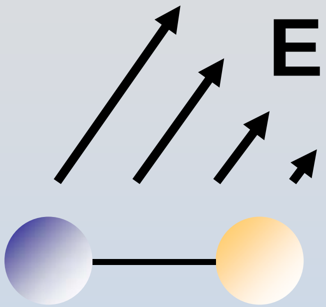


Animation (link)

- Field lines transmit tension
- Connection between dipole field and constant field “pulls” dipole into alignment

Concept Question Question: Dipole in Non-Uniform Field

Concept Question: Dipole in Non-Uniform Field



A dipole sits in a non-uniform electric field E, as shown

Due to the electric field this dipole will feel:

1. force but no torque
2. no force but a torque
3. both a force and a torque
4. neither a force nor a torque