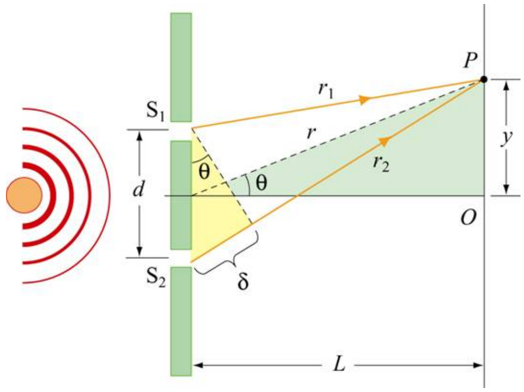
Diffraction
Challenge Problem Solutions

Problem 1:

Measuring the Wavelength of Laser Light

Suppose you shine a red laser through a pair of narrow slits (a = 40 μm) separated by a known distance and allow the resulting interference pattern to fall on a screen a distance L away (L ~ 40 cm).



(a) Will the center of the pattern (directly between the two holes) be an interference minimum or maximum?

(b) You should be able to easily mark and then measure the locations of the interference maxima. For the sizes given above, will these maxima be roughly equally spaced, or will they spread out away from the central peak? If you find that they are equally spaced, note that you can use this to your advantage by measuring the distance between distance maxima and dividing by the number of intermediate maxima to get an average spacing. If they spread out, which spacing should you use in your measurement to get the most accurate results, one close to the center or one farther away?

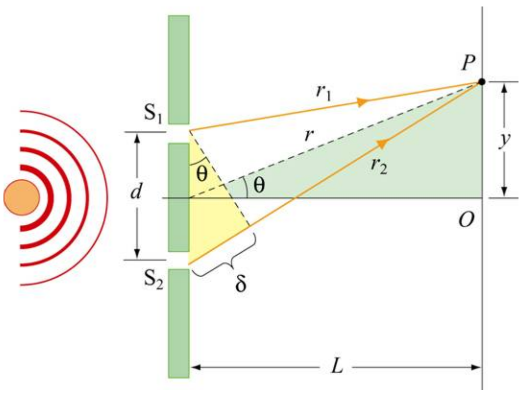
(c) Approximately how many interference maxima will you see on one side of the pattern before their intensity is significantly reduced by diffraction due to the finite width a of the slit?

(d) Derive an equation for calculating the wavelength λ of the laser light from your measurement of the distance Δy between interference maxima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab.

(e) In order to most accurately measure the distance between maxima Δy, it helps to have them as far apart as possible. (Why?) Assuming that the slit parameters and light wavelength are fixed, what can we do in order to make Δy bigger? What are some reasons that can we not do this ad infinitum?

Problem 1 Solutions:

(a) The center of the pattern will be a maximum because the waves from both slits travel the same distance to get to the center and hence are in phase.



(b) Looking at the picture at left, we get a maximum every time that the extra path length is an integral number of wavelengths:

$$d\sin\theta = m\lambda$$

The spacing is the distance between these locations, $y_{m+1} - y_m$. We can get y_m from θ:

$$\sin\theta_m = \frac{y_m}{\sqrt{L^2 + y_m^2}} = \frac{m\lambda}{d} \equiv \alpha_m \Rightarrow \frac{y_m^2}{L^2 + y_m^2} = \alpha_m^2$$

$$y_m^2\left(1-\alpha_m^2\right) = \alpha_m^2 L^2 \Rightarrow y_m = \frac{\alpha_m L}{\sqrt{1-\alpha_m^2}} \approx \alpha_m L\left(1-\frac{\alpha_m^2}{2}\right)$$

We have made the approximation that $\alpha_m \ll 1$, which is valid for the wavelengths and slit separations of this lab (it is order 10^{-3}). As long as this approximation is valid, we can also ignore the term that goes like $(\alpha_m)^2$, and hence we find the maxima are equally spaced:

$$y_{m+1} - y_m \approx \frac{\lambda L}{d}$$

(c) The first single slit minimum appears at $a\sin\theta = \lambda$. So when we approach:

$m = \frac{d}{\lambda}\sin\theta \approx \frac{d}{\lambda}\frac{\lambda}{a} = \frac{d}{a}$ we will lose signal due to the diffraction minimum.

(d) Using what we derived for part b,

$$\Delta y = y_{m+1} - y_m \approx \frac{\lambda L}{d} \quad \Rightarrow \quad \lambda = \frac{d\Delta y}{L}$$

(e) We can increase the distance to the screen and measure the distance between distant interference maxima (e.g. m = 1 and m = 4), which increases distances, making them easier to measure, and then allows us to divide down any measurement errors.

Problem 2:

Single Slit Interference

Now that you have measured the wavelength λ of the light you are using, you will want to measure the width of some slits from their diffraction pattern. When measuring diffraction patterns (as opposed to the interference patterns of problem 1) it is typically easiest to measure between diffraction minima.

(a) Derive an equation for calculating the width *a* of a slit from your measurement of the distance Δy between diffraction minima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab. Note that this same equation will be used for measuring the thickness of your hair.

(b) What is the width of the central maximum (the distance on the screen between the m=-1 and m=1 minima)? How does this compare to the distance Δy between other adjacent minima?

Problem 2 Solutions:

(a) Single slit minima obey the relationship $a\sin\theta = m\lambda$, which is the same formula as two slit maxima. So we can calculate the slit width from what we derived in 1b (replacing the distance between the slits *d* with the width of the single slit *a*):

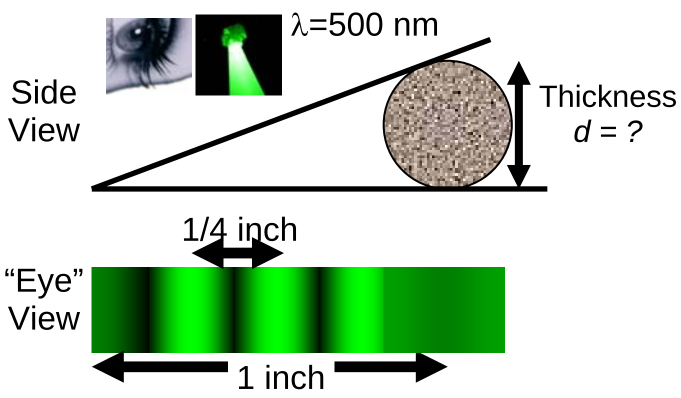
$$a = \frac{\lambda L}{\Delta y}$$

(b) The central minimum is twice as wide as the distance between other minima. It is:

$$\Delta y_{\text{central}} = 2\frac{\lambda L}{a}$$

Problem 3:

Another Way to Measure Hair

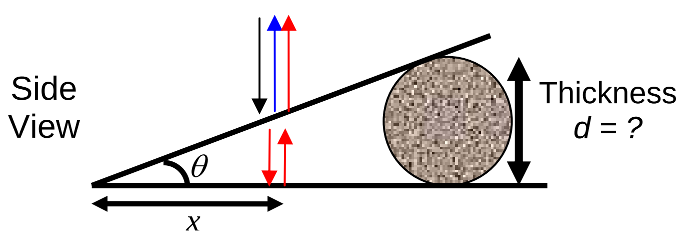


In addition to using hair as a thin object for diffraction, you can also measure its thickness using an interferometer. In fact, you can use this to measure even smaller objects. Its use on a small fiber is pictured at left. The fiber is placed between two glass slides, lifting one at an angle relative to the other. The slides are illuminated with green light from above, and when the set-up is viewed from above, an interference pattern, pictured in the "Eye View", appears.

What is the thickness *d* of the fiber?

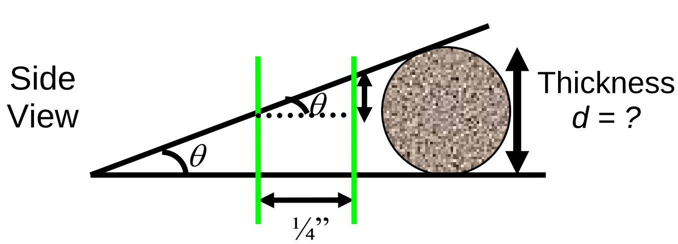
Problem 3 Solution:

The interference comes about because there are two paths the light can take. In the first light goes straight down, reflects off the glass, and goes straight back (we ignore the slight angle). In the second light goes down, passes through the glass and reflects off the lower glass, then goes straight back up. Let's redraw the picture as follows:



The light comes in (black arrow) and splits into two parts: immediate reflection (blue) and pass through then reflection (red). They eventually meet up to interfere. The extra path length taken by the second wave (red) is twice the height at that location, or $\delta = 2x\tan(\theta)$

Now consider two adjacent maxima, which apparently are about ¼ inch apart:



Notice that the extra height from the first to the second max (as indicated by the vertical arrow) is related to the distance between the successive maxima by:

$$\Delta h = \frac{\lambda}{2} = \frac{1}{4}\text{inch} \cdot \tan(\theta)$$

Why $\lambda/2$? Because the extra path (which is twice Δh) must be λ – one extra wavelength moves from one constructive maximum to the next. So:

$$d = 1\text{ inch} \cdot \left(\frac{\lambda}{2} \Big/ \frac{1}{4}\text{inch} \right) = 2\lambda = 1000\,\text{nm} = 1\,\mu\text{m}$$

Problem 4:

CD

Suppose you reflect light off of a CD and measure the resulting interference pattern on a screen a distance $L \sim 5$ cm away.

(a) A CD has a number of tracks, each of width d (this is what you are going to measure). Each track contains a number of bits, of length $l \sim d/3$. Approximately how many bits are there on a CD? In case you didn't know, CDs sample two channels (left and right) at a rate of 44100 samples/second, with a resolution of 16 bits/sample. In addition to the actual data bits, there are error correction and packing bits that roughly double the number of bits on the CD.

(b) What, approximately, must the track width be in order to accommodate this number of bits on a CD? In case you don't have a ruler, a CD has an inner diameter of 40 mm and an outer diameter of 120 mm.

(c) Derive an equation for calculating the width d of the tracks from your measurement of the distance Δy between interference maxima.

(d) Using the previous results, what approximately will the distance between interference maxima Δy be on the screen?

Problem 4 Solutions:

(a) A CD can store about 74 minutes of music, so:

$$\#\text{bits} \approx (74\text{ min})\left(60\frac{\text{s}}{\text{min}}\right)\left(44100\frac{\text{samp}}{\text{sec}}\right)\left(16\frac{\text{data bits}}{\text{samp}\cdot\text{chan}}\right)(2\text{ chan})\left(2\frac{\text{bits}}{\text{data bits}}\right)$$

$$\approx 12\times10^{9}\text{ bits}$$

(b) The track width d controls the number of tracks we end up with. What really matters is the overall length L of the tracks though. This is going to be a sum over the length of each track, starting with the inner most one (which has inner diameter ID = 40 mm) and going to the outer one (with outer diameter OD = 120 mm).

$$L = \sum_{\text{all tracks}} \ell_{\text{track}} = \sum_{n=0}^{N-1} \pi D_n = \pi \sum_{n=0}^{N-1}\left(ID + 2dn\right) = \pi\left(ID\cdot(N-1) + 2d\frac{(N-1)N}{2}\right)$$

$$= \pi(N-1)(ID + dN)$$

The number of tracks N is given by $N = (OD - ID)/2d$, so:

$$L = \pi\left(\frac{OD-ID}{2d} - 1\right)\left(\frac{OD+ID}{2}\right) \equiv \pi D_{ave}\left(\frac{\Delta r}{d} - 1\right) \approx \pi D_{ave}\frac{\Delta r}{d}$$

which makes sense – it's just the average diameter times the number of tracks.

No we can solve for the width d in terms of the # of bits that we need to store:

$$d \approx \pi D_{ave} \frac{\Delta r}{L} = \pi D_{ave} \frac{\Delta r}{(\#\ \text{bits})(\text{length } l/\text{bit})} = \pi D_{ave} \frac{\Delta r}{(\#\ \text{bits})(d/3)}$$

$$d \approx \sqrt{\pi D_{ave} \frac{3\Delta r}{(\#\ \text{bits})}} = \sqrt{\pi (80\ \text{mm}) \frac{3(40\ \text{mm})}{(12\times 10^9)}} \approx 1.6\ \mu\text{m}$$

I should comment that calling this distance the track width is a bit of a misnomer. More accurately, it is the distance between the tracks, which are only a few hundred nanometers wide.

(c) The derivation is just what we did in problems 1 and 2, yielding:

$$d = \frac{\lambda L}{\Delta y}$$

(d) I didn't tell you the wavelength of the light we will be using, but it's red so it's around $\lambda = 600$ nm, so

$$\Delta y = \frac{\lambda L}{d} \approx \frac{(600\ \text{nm})(5\ \text{cm})}{(1.6\ \mu\text{m})} \approx 2\ \text{cm}$$

Problem 5:

Light with a wavelength of $\lambda = 587.5$ nm illuminates a single slit which has a width of 0.750 mm.

(a) At what distance from the slit should a screen be located if the first minimum in the diffraction pattern is to be 0.850 mm from the center of the screen?

(b) What is the width of the central maximum?

Problem 5 Solution:

(a) The general condition for destructive interference is

$$\sin\theta = m\frac{\lambda}{a} \qquad m = \pm 1,\ \pm 2,\ \pm 3,\ ... \qquad (0.1)$$

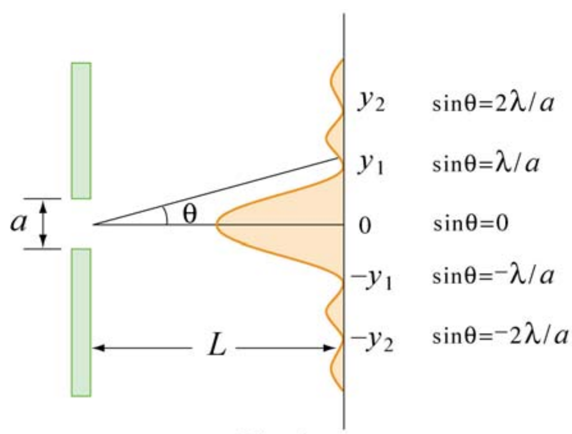
For small θ, we employ the approximation $\sin\theta \approx \tan\theta = y/L$, which yields

$$\frac{y}{L} \approx m\frac{\lambda}{a} \qquad (0.2)$$

The first minimum corresponds to $m = 1$. If $y_1 = 0.850 \text{ mm}$, then

$$L = \frac{ay_1}{m\lambda} = \frac{(7.50\times10^{-4}\text{ m})(8.50\times10^{-4}\text{ m})}{1(587.5\times10^{-9}\text{ m})} = 1.09\text{ m} \qquad (0.3)$$

The diffraction pattern is shown in the figure below.



Viewing screen

(b) As seen from the figure above, the width of the central maximum is

$$w = 2y_1 = 2\left(0.850\times 10^{-3}\ \text{m}\right) = 1.70\ \text{mm}\ (0.4)$$

Problem 6:
In a single-slit diffraction, the second-order bright fringe is at a distance1.40 mm from the center of the central maximum. The screen is 80.0 cm from a slit of width 0.800 mm. Assuming that the incident light is monochromatic, calculate the approximate wavelength of the incident light.

Problem 6 Solution:

The general condition for destructive interference is

$$\sin\theta = m\frac{\lambda}{a} \approx \frac{y}{L} \qquad (1.1)$$

where small-angle approximation has been made. Thus, the position of the *m*-th order dark fringe measured from the central axis is

$$y_m = m\frac{\lambda L}{a} \qquad (1.2)$$

Let the second bright fringe be located halfway between the second and the third dark fringes, i.e.,

$$y_2' = \frac{1}{2}(y_2 + y_3) = \frac{1}{2}(2+3)\frac{\lambda L}{a} = \frac{5\lambda L}{2a} \qquad (1.3)$$

The approximate wavelength of the incident light is then

$$\lambda \approx \frac{2ay_2'}{5L} = \frac{2(0.800\times10^{-3}\ \text{m})(1.40\times10^{-3}\ \text{m})}{5(0.800\ \text{m})} = 5.60\times10^{-7}\ \text{m} \qquad (1.4)$$

Problem 7:

Coherent light with a wavelength of $\lambda = 501.5$ nm is sent through two parallel slits in a large flat wall. Each slit has a width $a = 0.700\ \mu\text{m}$, and the centers of the slits are at a distance $d = 2.80\ \mu\text{m}$ apart. The light falls on a semi-cylindrical screen, with its axis at the midline between the slits.

(a) Predict the direction of each interference maximum on the screen, as an angle away from the bisector of the line joining the slits.

(b) Describe the pattern of light on the screen, specifying the number of bright fringes and the location of each.

(c) Find the intensity of light on the screen at the center of each bright fringe, expressed as a fraction of the light intensity I_0 at the center of the pattern.

Problem 7 Solutions:

(a) The condition for double-slit interference maxima is given by

$$d \sin\theta = m\lambda \qquad m = 0,\ \pm 1,\ \pm 2,... \qquad (2.1)$$

which yields

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) \qquad (2.2)$$

With $\lambda = 5.015\times 10^{-7}\,\text{m}$ and $d = 2.80\times 10^{-6}\,\text{m}$, the above equation becomes

$$\theta_m = \sin^{-1}\left(m\ \frac{5.015\times 10^{-7}\,\text{m}}{2.80\times 10^{-6}\,\text{m}}\right) = \sin^{-1}(0.179m) \qquad (2.3)$$

The solutions are

$$
\begin{aligned}
\theta_0 &= 0^\circ \\
\theta_{\pm1} &= \sin^{-1}(\pm 0.179) = \pm 10.3^\circ \\
\theta_{\pm2} &= \sin^{-1}(\pm 0.358) = \pm 21.0^\circ \\
\theta_{\pm3} &= \sin^{-1}(\pm 0.537) = \pm 32.5^\circ \\
\theta_{\pm4} &= \sin^{-1}(\pm 0.716) = \pm 45.7^\circ \\
\theta_{\pm5} &= \sin^{-1}(\pm 0.895) = \pm 63.5^\circ \\
\theta_{\pm6} &= \sin^{-1}(\pm 1.074) = \text{ no solution}
\end{aligned} \qquad (2.4)
$$

Thus, we see that there are a total of 11 directions of interference maxima.

(b) The general condition for single-slit diffraction minima is $a\sin\theta = m\lambda$, or

$$
\theta_m = \sin^{-1}\left(\frac{m\lambda}{a}\right) \qquad m = \pm 1,\ \pm 2,\ \ldots \qquad (2.5)
$$

With $\lambda = 5.015\times 10^{-7}\,\text{m}$ and $a = 7.00\times 10^{-7}\,\text{m}$, the above equation becomes

$$
\theta_m = \sin^{-1}\left(m\ \frac{5.015\times 10^{-7}\,\text{m}}{7.00\times 10^{-7}\,\text{m}}\right) = \sin^{-1}\left(0.716m\right) \qquad (2.6)
$$

The solutions are

$$
\begin{aligned}
\theta_{\pm1} &= \sin^{-1}\left(\pm 0.716\right) = \pm 45.7^\circ \\
\theta_{\pm2} &= \sin^{-1}\left(\pm 0.1.43\right) = \text{ no solution}
\end{aligned} \qquad (2.7)
$$

Since these angles correspond to dark fringes, the total number of bright fringes is $N = 11 - 2 = 9$.

(c) The intensity on the screen is given by

$$
I = I_0\left[\frac{\sin\left(\pi a\sin\theta/\lambda\right)}{\pi a\sin\theta/\lambda}\right]^2 \qquad (2.8)
$$

where I_0 is the intensity at $\theta = 0$.

(i) At $\theta = 0^\circ$, we have $\dfrac{\sin\theta}{\theta} \to 1$ which yields $\dfrac{I}{I_0} = 1.00$

(ii) At $\theta = \pm 10.3°$, we have $\dfrac{\pi a \sin\theta}{\lambda} = \pm\dfrac{\pi(0.700\ \mu\text{m})\sin 10.3°}{0.5015\ \mu\text{m}} = \pm 0.785\ \text{rad} = \pm 45.0°$, which gives

$$\frac{I}{I_0} = \left[\pm\frac{\sin 45.0°}{0.785}\right]^2 = 0.811$$

(iii) At $\theta = \pm 21.0°$, we have $\dfrac{\pi a \sin\theta}{\lambda} = \pm 1.57\ \text{rad} = \pm 90.0°$, and the intensity ratio is

$$\frac{I}{I_0} = \left[\pm\frac{\sin 90.0°}{1.57}\right]^2 = 0.406$$

(iv) At $\theta = \pm 32.5°$, we have $\dfrac{\pi a \sin\theta}{\lambda} = \pm 2.36\ \text{rad} = \pm 135°$, and the intensity ratio is

$$\frac{I}{I_0} = \left[\pm\frac{\sin 135°}{2.36}\right]^2 = 0.0901$$

(v) At $\theta = \pm 63.5°$, we have $\dfrac{\pi a \sin\theta}{\lambda} = \pm 3.93\ \text{rad} = \pm 225°$, and the intensity ratio is

$$\frac{I}{I_0} = \left[\pm\frac{\sin 225°}{3.93}\right]^2 = 0.0324$$