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18.085 Computational Science and Engineering I
Fall 2008

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Your name is:	_____	Grading	1 (36)
			2 (36)
			3 (28)
		Total	_____

In Questions 1 and 2, part (a) is checked by a later part and Question 3 is not hard.

Thank you for taking 18.085. I appreciate the good homeworks and consistent effort.

I hope very much that you will find this subject useful.

(1a) GRAPH the 2π -periodic $f(x)$ defined between $-\pi$ and π by

$$f(x) = e^x \text{ for } -\pi \leq x \leq 0, \quad f(x) = e^{-x} \text{ for } 0 \leq x \leq \pi.$$

Find the Fourier coefficients c_k in $f(x) = \sum c_k e^{ikx}$ directly from the standard integral formula.

(1b) Draw the graphs of df/dx and d^2f/dx^2 between $-\pi$ and π . What rate of decay for the coefficients c_k ?

(1c) Find the same coefficients c_k in a different way, starting from

$$-\frac{d^2f}{dx^2} + f(x) = \text{what function } R(x)?$$

With $f(x) = \sum c_k e^{ikx}$ take Fourier coefficients of both sides of this equation and solve for c_k .

(2a) Find the *cyclic* convolution $\mathbf{w} = \mathbf{u} * \mathbf{v}$ ($N = 4$) of the vectors $\mathbf{u} = (1, 0, 1, 0)$ and $\mathbf{v} = (0, 1, 0, 1)$.

(2b) Find the discrete Fourier coefficients c_k and d_k (4-point DFT) of those vectors \mathbf{u} and \mathbf{v} respectively. From the c_k and d_k find the Fourier coefficients h_k of their convolution $\mathbf{w} = \mathbf{u} * \mathbf{v}$. How does this answer confirm that \mathbf{w} in part (a) was correct??

(2c) For some 4-component vectors \mathbf{z} the cyclic convolution with $\mathbf{u} = (1, 0, 1, 0)$ is $\mathbf{u} * \mathbf{z} = (0, 0, 0, 0)$. Describe the components of $\mathbf{z} = (z_0, z_1, z_2, z_3)$ **and also** its Fourier coefficients (C_0, C_1, C_2, C_3) .

(3a) Find the Fourier integral transform $\widehat{f}(k)$ of the square wave $f(x)$ between -1 and 1 :

$$f(x) = 1 \text{ for } 0 \leq x \leq 1, \quad f(x) = -1 \text{ for } -1 \leq x < 0, \quad f(x) = 0 \text{ for } |x| > 1.$$

(3b) Find the Fourier integral transform \widehat{D} of the *derivative* of that square wave. Also find the transform \widehat{C} of the *convolution* of that square wave with its derivative. Factors of 2π are forgiven ...

