MATH 31A (Butler)
Practice for Midterm IIa

Try to answer the following questions without the use of book, notes or calculator. Time yourself and try to finish the questions in less than 50 minutes.

1. (a) Verify that $(2, 1)$ is a critical point for the curve $y^3x - 3yx^2 = x^2 - 15x + 16$. (You need to verify two things: (i) it is on the curve and (ii) it is a critical point.)

(b) Use the second derivative test to determine if the point $(2, 1)$ is a maximum or a minimum.

2. You have recently started a new job as quality control officer in the new widget factory, company slogan: "You fidget, we widget". A new machine has been installed that takes a rod of length x inches for x between 1 and 10 and returns a widget of size $x^2 + 3x + 2$. Ideally, you want all of your widgets to have an output of size 20, which would correspond to starting with a rod of exactly 3 inches, but you know that the rods are not exactly 3 inches.

Given that you want the widgets to be within $\frac{1}{2}$ of 20 (i.e., you can have widgets of sizes in the range of $20 \pm \frac{1}{2}$), estimate how much error you can have for the length of the rods that go in.

3. (a) After several months as quality control officer you have been promoted and are now an executive. After changing the company slogan to "Don't be a nidget! Get the best widget!", you have been asked to decide how many widgets to produce in order to maximize the profit (which is the difference between revenue and cost).

If w is the number of widgets then the cost of producing the widgets is $90 + \frac{1}{10}w^2$ dollars and each widget can be sold for for \$8, how many widgets should you produce, and how much profit will the company make?

(b) At what price for widget will the company only be able to break even at best?

4. (a) On what intervals is $h(x) = x^2 - 2\arctan(x^2)$ increasing and on what intervals is it decreasing?

(b) Find the global maximum and global minimum for $h(x) = x^2 - 2\arctan(x^2)$ on the interval $0 \le x \le \sqrt[4]{3}$. (Hint: $h(\sqrt[4]{3}) = \sqrt{3} - \frac{2}{3}\pi \approx -0.362344$.)

5. Consider the functions $f(x) = 2\tan x + 2\sec x$ and $g(x) = (\tan x + \sec x)^2 + 1$. Is $g(x)$ the derivative of $f(x)$ or is $g(x)$ the anti-derivative of $f(x)$? Justify your answer. (Hint: $\sec^2 x = \tan^2 x + 1$.)