

MATH 31A (Butler)

Practice for Midterm I

Try to answer the following questions without the use of book, notes or calculator. Time yourself and try to finish the questions in less than 50 minutes.

1. (a) Find $\lim_{t \rightarrow 0} \frac{\left(\frac{1}{\sqrt{9+2t}} - \frac{1}{3}\right)}{\sin(3t)}$.

(b) Find $\lim_{x \rightarrow 0} \left(\frac{1}{3x} - \frac{1}{x(x+3)}\right)$.

2. Let $g(x) = \begin{cases} x \cos x - 2x + 3 & \text{if } x \leq 0; \\ bx + 3 & \text{if } x > 0. \end{cases}$

(a) Show that $g(x)$ is continuous at $x = 0$ for *any* value of b .

(b) For what value of b does $g'(0)$ exist?

(c) For the answer in part (b), what is the value of $g'(0)$?

3. Find the tangent line of $y = 2x + 4\sqrt{x} - \pi^2$ parallel to the line $y = 3x + 7$.

4. Express $\frac{d^2}{dx^2}(f(x)g(x))$ in terms of $f(x)$, $f'(x)$, $f''(x)$, $g(x)$, $g'(x)$ and $g''(x)$.

5. Given that $f(1) = 2$, $f'(1) = -1$, $g(1) = 3$ and $g'(1) = 2$ find $h(1)$ and $h'(1)$ where $h(x) = x^2 f(x) - 3\sqrt{x}g(x)$.

6. The position of a particle on a strip is given by $s(t) = t^3 - 5t^2 + 6t - 4$ ($s(t)$ is measured in inches and t is measured in seconds).

(a) What is the position of the particle at $t = 1$? (Give the units.)

(b) What is the (instantaneous) velocity of the particle at $t = 1$? (Give the units.)

(c) What is the acceleration of the particle at $t = 1$? (Give the units.) Does this mean the speed the particle is moving is going up or down?