

**MATH 31A (Butler)**  
Practice for Final (C)

*Try to answer the following questions without the use of book, notes or calculator; but you can use the equation sheet posted on the course website. Time yourself and try to finish the questions in less than three hours.*

1. Find the area of the region bounded between the curves  $g(t) = \sin(\pi t)$  and  $f(t) = 8t^2 - 2t$ . (Hint: the best way to find the intersection points is to “guess” some easy values for  $t$ .)

2. (a) For  $a \geq \frac{1}{2}$  find the point on the curve  $y = \sqrt{x}$  closest to the point  $(a, 0)$ .

(b) What is the closest point on the curve  $y = \sqrt{x}$  for the point  $(a, 0)$  when  $a < \frac{1}{2}$ ?

3. To help practice your timing on the final you have started using a large hourglass when doing practice problems. But before long you discover that it is much more interesting to watch the hourglass than to work on the practice problems! In particular, you notice that the sand in the top half of the hourglass forms a cone shape where sand drops out of the tip of the cone into the bottom half of the hourglass. By repeated playing around with the sand you know that the sand drops out of the top half of the hourglass at a constant rate of  $2\pi$  cubic inches per minute. You also see that when you first flip the hourglass over that the cone formed by the sand has a height of five inches and is four inches across at the top.

At what height will the depth of the sand be when the sand is dropping at a rate of two inches per minute? (Hint: the volume of a cone is  $V = \frac{1}{3}\pi r^2 h$  where  $r$  is the radius and  $h$  is the height.)

4. (a) Find the three  $y$ -intercepts of the implicitly defined curve  $y^3 + 4 \sin(xy) = y + 5x$ .  
(b) For each point found in part (a) find the tangent line to the curve.

5. (a) Find  $\int x(\sin(x^2) + \cos(x^2))^2 \sin(x^2) dx$ .

(b) Find  $\int \frac{1-x^2}{1+x^2} dx$ . (Hint:  $1-x^2 = 2 - (1+x^2)$ .)

6. Given that  $\int_3^x g(t) dt = \sqrt[3]{x^2 - 1} + Cx$ , find  $C$  and  $g(x)$ .

7. In the ten weeks leading up to the final you notice that the amount of time spent studying for this class has been increasing. In particular, you see that on the  $t$ th week of the quarter you were studying for  $2 + t/5$  hours per week. Using integration, find the total amount of time that you spent studying for the final (i.e. from  $t = 0$  weeks to  $t = 10$  weeks).

8. Find the average value for  $f(x) = |4 - x^2|$  for the interval  $-3 \leq x \leq 5$ .

9. (a) Use substitution to show that for a function  $f(x)$  that

$$\int_0^a \frac{f(x)}{f(x) + f(a-x)} dx = \int_0^a \frac{f(a-x)}{f(x) + f(a-x)} dx.$$

(b) Show that

$$\int_0^a \frac{f(x)}{f(x) + f(a-x)} dx = \frac{1}{2}a.$$

(Hint:  $\int_0^a g(x) dx = \frac{1}{2}(\int_0^a g(x) dx + \int_0^a g(x) dx)$ , and use part (a).)

(c) Find  $\int_0^{\pi/2} \frac{1}{1 + \tan x} dx$ . (Hint:  $\sin x = \cos(\frac{\pi}{2} - x)$ .)

10. Find the volume created when the region in the first quadrant bounded by the  $x$ -axis, the  $y$ -axis and  $y = \sqrt{1 - x^4}$  is rotated

(a) around the  $x$ -axis.

(b) around the  $y$ -axis. (Hint:  $1 - x^4 = 1 - (x^2)^2$ .)