MATH 31A (Butler)
Practice for Final (B)

Try to answer the following questions without the use of book, notes or calculator; but you can use the equation sheet posted on the course website. Time yourself and try to finish the questions in less than three hours.

1. Show that $\int_0^{\pi/4} \tan x \, dx + \int_0^1 \arctan x \, dx = \frac{\pi}{4}$. (Hint: interpret each integral as an area and show how the areas "piece" together.)

2. A particle moves along the curve implicitly defined by $xy^4 - yx^4 = x - y^2$. When the particle passes through the point $(1, 1)$ its x coordinate is changing $1/4$ units per second. How fast is the y coordinate changing?

3. The night before the final you have decided to do one last study session. But before you begin you decide that you want to make the best use of your time. You know that there is a diminishing return to the amount of time you study (i.e., you get more out of your first hour of study than you will your second; and more out of your second hour than you will your third). At the same time you know that the longer you study the less sleep you will have and the harder it will be to concentrate on the test (which will make problems about optimizing your study session even harder!). After thinking about it for a few minutes you decide if H is the number of hours that you study that night then you anticipate your score on the final will be

$$115 - \frac{50}{H+1} - 8H,$$

more or less. How many hours should you study to maximize your score, and what should your anticipated score on the final be?

4. Use the following information to get an estimate for $g(f(2.1))$.

x	0	1	2	3
$f(x)$	-1	3	0	2
$f'(x)$	1	-2	3	0
$g(x)$	3	0	1	2
$g'(x)$	1	3	2	2

5. (a) Find $\int (\sec\theta + \tan\theta)^2 \, d\theta$.

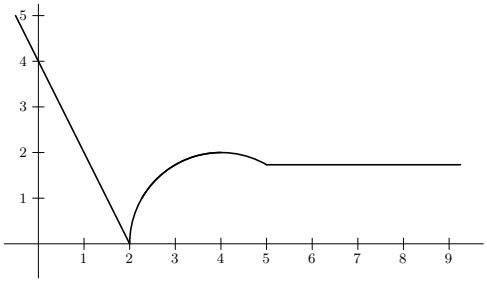
(b) Find $\int \frac{x}{\sqrt{x^2+1} + x} \, dx$.

6. Suppose that $y''(t) = 2 - \sin(\pi t)$ and that $y(0) = -2$ and $y(2) = 5$. What is $y'(1)$?

7. Let $H(x) = \int_{x^2}^{x+2} h(t)\,dt$ where $h(t)$ is the function defined piecewise by

$$h(t) = \begin{cases} 4 - 2t & \text{if } t \le 2, \\ \sqrt{4 - (t-4)^2} & \text{if } 2 \le t \le 5, \\ \sqrt{3} & \text{if } t \ge 5. \end{cases}$$

A graph of this function is shown on the right. Find the following values.



(a) $H(-2) =$
(b) $H(0) =$
(c) $H'(1) =$
(d) $H(2) =$
(e) $H(3) =$

8. What is the average area of circles where the radii of the circles can range from 1 to 4?

9. Let $h(x)$ be a function such that

$$\int_0^2 h(x)\,dx = 1,\ \int_0^3 h(x)\,dx = 2,\ \int_0^4 h(x)\,dx = 6,\ \int_1^5 h(x)\,dx = 5,\ \text{and}\ \int_2^5 h(x)\,dx = 7.$$

Find $\int_1^3 h(x)\,dx$.

10. A torus (or as Homer Simpson would say "mmmmm, donut") can be formed by spinning a circle around the x-axis. Find the volume of the torus found by spinning the circle of radius 1 centered at $(0, 2)$ around the x-axis.
(Hint: the curve describing the top of the circle is $y = 2 + \sqrt{1 - x^2}$ while the curve describing the bottom of the circle is $y = 2 - \sqrt{1 - x^2}$.)