3. Linear Potential Theory
Useful weblinks:
- Blackboard
- www.rwpoll.com

Written exam

OE4630 module II course content

- +/- 7 Lectures
- Bonus assignments (optional, contributes 20% of your exam grade)
- Laboratory Exercise (starting 30 nov)
  - 1 of the bonus assignments is dedicated to this exercise
  - Groups of 7 students
  - Subscription available soon on BB
- Written exam

Written exam

Teacher module II:
- Ir. Peter Naaijen
- p.naaijen@tudelft.nl
- Room 34 B-D-360 (next to towing tank)

Book:
- Offshore Hydromechanics, by J.M.J. Journee & W.W. Massie

Useful weblinks:
- http://www.shipmotions.nl
- Blackboard
### Lecture notes:

- **Disclaimer:** Not everything you (should) learn is in the lecture notes (lees: niet alles wat op het tentamen gevraagd kan worden staat in diktant...) -7

Make personal notes during lectures!!

- Don’t save your questions till the break -7

Ask if anything is unclear

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### Introduction

**Offshore oil resources have to be explored in deeper water floating structures instead of bottom founded**
Reasons to study waves and ship behavior in waves:

- Determine allowable / survival conditions for offshore operations
- Direct wave induced structural loads
- Minimum required air gap to avoid wave damage
- Motion Analysis
- Decommissioning / Installation / Pipe laying - Excalibur / Allseas ‘Pieter Schelte’
Introduction

Reasons to study waves and ship behavior in waves:
• the dynamic loads on the floating structure, its cargo or its equipment:
  - Forces on mooring system, motion envelopes loading arms

Floating Offshore: More than just oil

Wave energy conversion

Floating wind farm

OTE

Mega Floaters
Introduction

Reasons to study waves and ship behavior in waves:
- Determine allowable / survival conditions for offshore operations
- Downtime analysis

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Wave Height (m)</th>
<th>Wave Period (s)</th>
<th>Wave Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>0.2</td>
<td>1.0</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.5</td>
<td>2.5</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>1.0</td>
<td>5.0</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>2.0</td>
<td>10.0</td>
<td>100</td>
<td>100</td>
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Definitions & Conventions

Regular waves
Ship motions

Real-time motion prediction
Using X-band radar remote wave observation

apparently irregular but can be considered as a superposition of a finite number of regular waves, each having own frequency, amplitude and propagation direction
• Regular waves

regular wave propagating in direction $\mu$:

$$ (t, x) = \cos t \ k x \cos ky \sin k \ t \ \frac{1}{T} $$

Linear solution Laplace equation

Regular waves

regular wave propagating in direction $\mu$:

$$ (t, x) = \cos t \ k x \cos ky \sin k \ t \ \frac{1}{T} $$

Co-ordinate systems

Definition of systems of axes

Earth fixed: ($x_0, y_0, z_0$)

wave direction with respect to ship’s axes system:

Wave direction = $x_0$
Behavior of structures in waves
Ship's body bound axes system \((x_b, y_b, z_b)\) follows all ship motions

Definition of rotations

<table>
<thead>
<tr>
<th></th>
<th>NE</th>
<th>EN</th>
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<tbody>
<tr>
<td>1</td>
<td>x</td>
<td>Schrikken</td>
</tr>
<tr>
<td>2</td>
<td>y</td>
<td>Verzetten</td>
</tr>
<tr>
<td>3</td>
<td>z</td>
<td>Domen</td>
</tr>
</tbody>
</table>

How do we describe ship motion response?
Rao's Phase angles
Mass-Spring system:

\[ m \ddot{z} + c \dot{z} + F_x \cos t = 0 \]  
\[ \text{Motion equation} \]

\[ z = z_0 \cos t \]  
\[ \text{Steady state solution} \]

Motions of and about COG

- **Surge** (schrikken): \[ x = x_0 \cos t \]
- **Sway** (verzetten): \[ y = y_0 \cos t \]
- **Heave** (dompen): \[ z = z_0 \cos t \]
- **Roll** (rollen): \[ \phi = \phi_0 \cos t \]
- **Pitch** (stampen): \[ \theta = \theta_0 \cos t \]
- **Yaw** (gieren): \[ \psi = \psi_0 \cos t \]

Phase angles are related to undisturbed wave at origin of steadily translating ship-bound system of axes (COG).

RAO and phase depend on:
- Wave frequency
- Wave direction
Example: roll signal

\[ \phi(t) = \phi_0 \cos(\omega t + \omega_0 t) \]

Displacement: \( a \cos \omega t \)
Velocity: \( e \sin \omega t - e \cos \omega t / 2 \)
Acceleration: \( 2e \cos \omega t + 2e \cos \omega t / 2 \)

Consider long waves relative to ship dimensions

What is the RAO of pitch in head waves?

- Phase angle heave in head waves?
- RAO pitch in head waves?
- Phase angle pitch in head waves?
- Phase angle pitch in following waves?

Local motions (in steadily translating axis system)

- Only variations:
- Linearized:

\[
\begin{bmatrix}
Y_p & Y_t & t & 0 & t & Y_{SP} \\
Z_p & Z_t & t & 0 & t & Z_{SP} \\
\end{bmatrix}
\]

Location considered point

\[
\begin{bmatrix}
X_p & Y_p & Z_p \\
Y_p & Y_t & Y_{SP} \\
Z_p & Z_t & Z_{SP} \\
\end{bmatrix}
\]
Local Motions

Example 3: horizontal crane tip motions

The tip of an onboard crane, location: $x_b, y_b, z_b = -40, -9.8, 25.0$

For a frequency $\omega = 0.6$ the RAO's and phase angles of the ship motions are:

<table>
<thead>
<tr>
<th>Motion</th>
<th>RAO</th>
<th>Phase</th>
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<tr>
<td>Surge</td>
<td>$x$</td>
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<td>Heave</td>
<td>$z$</td>
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<tr>
<td>Roll</td>
<td>$\phi$</td>
<td>$\cos t$</td>
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<td>$\cos t$</td>
</tr>
<tr>
<td>Yaw</td>
<td>$\psi$</td>
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Calculate the RAO and phase angle of the transverse horizontal motion ($y$-direction) of the crane tip.

Relation between Motions and Waves

How to calculate RAO's and phases?

Input: regular wave, $\omega$

Output: regular motion $\omega$, RAO, phase

Mass-Spring system:

Forces acting on body:

$F$...

Complex notation of harmonic functions

$1$ Surge ($\text{schrikken}$): $x \cdot \cos t$

$\Re x e^{i\omega t}$

$\Re x e^{i\omega t}$

Complex motion amplitude

$\Re x e^{i\omega t}$

For a frequency $\omega = 0.6$ the RAO's and phase angles of the ship motions are:

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OE4630 2012-2013, Offshore Hydromechanics, Part 2

Marine Engineering, Ship Hydromechanics Section
Mass-Spring system:

\[ m \ddot{z} + b \dot{z} + c_z z = F \cos(\omega t) \]

**Transient solution**

\[ z(t) = A e^{\xi t} \sin \left( \sqrt{\xi^2 - \zeta^2} \omega t + \varphi \right) \]

\[ \xi = \frac{b}{2mc} \]

**Damping ratio**

\[ \zeta = \frac{b}{2mc} \]

**Steady state solution:**

\[ z(t) = z_0 \cos(\omega t + \varphi) \]

\[ E = a \tan \left( \frac{(-b)}{-4b + (m + c)} \right) \]

\[ z_0 = \frac{E}{\sqrt{(m + c)^2 + (4b)^2}} \]

---

**Moving ship in waves:**

\[ m_z z + b_z \dot{z} + c_z z = F_{za} \cos(\omega t) \]

Restoring coefficient for **heave**

\[ m_y \ddot{\phi} + b_y \dot{\phi} + c_y \dot{\phi} = F_{ya} \cos(\omega t) \]

Restoring coefficient for **roll**

---

**What is the hydrostatic spring coefficient for the sway motion?**

\[ m_y y + b_y \dot{y} + c_y \dot{y} = F_z \cos(\omega t) \]

A. \( c_z = A_{wl} \rho g \)
B. \( c_z = A_{lat} \rho g \)
C. \( c_z = 0 \)

---

**Non linear stability issue...**
Roll restoring

Roll restoring coefficient:

\[ c_4 = \rho g v \cdot GM \]

What is the point the ship rotates around statically speaking? (Ch 2)

Moving ship in waves:

\[ m_1 \dot{\phi} + b_1 \phi + c_1 \phi = F_1 \cos(\omega t) \]

Restoring coefficient for roll?

\[ \dot{z} = FG - FG \cos \varphi = 0 \]

\[ \ddot{y} = FG \sin \varphi = FG \dot{\varphi} \]

Floating stab.

Stability moment

\[ M = \rho g V \cdot GM \cdot \varphi \]

Moving ship in waves:

Not in air but in water!

\[ F = m \ddot{z} \]

\[ F_c = -c \cdot z \]

\[ -b \cdot \ddot{z} \]

(Only potential + wave damping)

\[ (m + a) \ddot{z} + b \cdot \ddot{z} + c \cdot z = F_c \]
Moving ship in waves:

Analogy / differences with mass-spring system:

| External force | F(t) | Wave exciting force
<table>
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<tr>
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<tbody>
<tr>
<td>restoring force</td>
<td>c^2</td>
<td>Archimedes buoyancy</td>
</tr>
<tr>
<td>Damping force</td>
<td>b dz/dt</td>
<td>Hydrodynamic damping</td>
</tr>
<tr>
<td>Inertia force</td>
<td>m^2 dz/dt</td>
<td>Mass = Hydrodynamic Mass</td>
</tr>
</tbody>
</table>

Depend on frequency!

\[
(m + a) \ddot{z} + b \dot{z} + c \cdot z = F_w
\]

Moving ship in waves:

Right hand side of m.e.:
Wave Exciting Forces

- incoming: regular wave with given frequency and propagation direction
- Assuming the vessel is not moving
Back to Regular waves

regular wave propagating in direction $\mu$

$$\zeta (t, x) = \zeta_e \cos \left( \omega t - k x \cos \mu - k y \sin \mu \right)$$

Linear solution Laplace equation

In order to calculate forces on immersed bodies: What happens underneath free surface?

Potential Theory

What is potential theory? A way to give a mathematical description of flowfield.

Most complete mathematical description of flow is viscous Navier-Stokes equation:

Navier-Stokes vergelijkingen:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g}$$

(not relaxed)
Water is hard to compress, we will assume this is impossible

→

Apply principle of continuity on control volume:

![Diagram of control volume]

Continuity: what comes in, must go out.

This results in continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

If in addition the flow is considered to be irrotational and non-viscous →

Velocity potential function can be used to describe water motions

Main property of velocity potential function:

for potential flow, a function \( \Phi(x,y,z,t) \) exists whose derivative in a certain arbitrary direction equals the flow velocity in that direction. This function is called the velocity potential.

From definition of velocity potential:

\[
u = \frac{\partial \Phi}{\partial x}, \quad v = \frac{\partial \Phi}{\partial y}, \quad w = \frac{\partial \Phi}{\partial z}
\]

Substituting in continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

Results in Laplace equation:

\[
\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0
\]
Summary

- Potential theory is a mathematical way to describe flow.

Important facts about the velocity potential function $\Phi$:
- Definition: $\Phi$ is a function whose derivative in any direction equals the flow velocity in that direction.
- $\Phi$ describes non-viscous flow.
- $\Phi$ is a scalar function of space and time (NOT a vector!)

Water Particle Kinematics

Trajectories of water particles in infinite water depth

$\Phi(x, y, z, t) = \frac{\zeta}{\omega} \cdot e^{\omega t} \cdot \sin(kx \cos \mu + ky \sin \mu - \omega t)$

Summary

- Velocity potential for regular wave is obtained by
  - Solving Laplace equation satisfying:
    1. Seabed boundary condition
    2. Dynamic free surface condition

$\Phi(x, y, z, t) = \frac{\zeta}{\omega} \cdot e^{\omega t} \cdot \sin(kx \cos \mu + ky \sin \mu - \omega t)$

$\Phi(x, y, z, t) = \frac{\zeta}{\omega} \cdot \cosh(k(h + z)) \cdot \sin(kx \cos \mu + ky \sin \mu - \omega t)$

3. Kinematic free surface boundary condition results in:

- Laplace equation = relation between wave frequency and wave length

$\omega^2 = kg \tanh(kh)$

Water Particle Kinematics

Trajectories of water particles in finite water depth

$\Phi(x, y, z, t) = \frac{\zeta}{\omega} \cdot \cosh(k(h + z)) \cdot \sin(kx \cos \mu + ky \sin \mu - \omega t)$

$\Phi(x, y, z, t) = \frac{\zeta}{\omega} \cdot \frac{\cosh(kh)}{\cosh(k(h + z))} \cdot \sin(kx \cos \mu + ky \sin \mu - \omega t)$
Pressure

Pressure in the fluid can be found using Bernoulli equation for unsteady flow:

\[ \frac{\partial \Phi}{\partial t} + \frac{1}{2} (u^2 + w^2) + \frac{p}{\rho} + gz = 0 \]

\[ p = -\rho \frac{\partial \Phi}{\partial t} + \frac{1}{2} \rho (u^2 + w^2) - \rho gz \]

1st order fluctuating pressure
2nd order (small quantity squared = small enough to neglect)
Hydrostatic pressure (Archimedes)

Potential Theory

- Pressure
- Forces and moments can be derived from pressures:

\[ MT = \iiint_S p \cdot (\mathbf{r} \times \mathbf{n}) \, dS \]

Which structure experiences the highest vertical wave load acc. to potential theory?

A. The Barge
B. The Spar
C. Equal forces on both
Flow superposition

1. Flow due to Undisturbed wave
   \( \Phi_u = -\frac{Z}{\omega} e^{i \omega t} \sin(\omega t - kx \cos \mu - ky \sin \mu) \)

Flow due to Diffraction

Has to be solved. What is boundary condition at body surface?

Pressure due to undisturbed incoming wave

\( T = 4 \) s

Exciting force due to waves

\( (m + a)b + \kappa c \cdot z = F_u \)

1. Undisturbed wave force (Froude-Krilov)
   \( \Phi_u = -\frac{Z}{\omega} e^{i \omega t} \sin(\omega t - kx \cos \mu - ky \sin \mu + \epsilon) \)

2. Diffraction force

Has to be solved. What is boundary condition at body surface?

Pressure due to undisturbed incoming wave

\( T = 10 \) s
Wave Forces
Wave force acting on vertical wall

Force on the wall
\[ F = \int \rho \, \omega \, \cos(\omega) \, dx \]
\[ \Phi = \int \frac{\omega}{\nu} \, \sin(\nu x) \, dx \]
\[ \omega = \int \left( \frac{\omega}{\nu} \, \sin(\nu x) \right) \, dx \]
\[ \Phi = \int \frac{\omega}{\nu} \, \sin(\nu x) \, dx \]

Calculating hydrodynamic coefficient and diffraction force
\[ (m + a) \cdot \Phi = \int c \cdot \omega \, \sin(\omega) \, dx = F_w = F_{1x} + F_{2x} \]
\[ \Phi = \int \frac{\omega}{\nu} \, \sin(\nu x) \, dx \]

Left hand side of m.e.:
Hydromechanic reaction forces
- NU incoming waves:
- Vessel moves with given frequency
Recap: Motion equation

\[(m + a) \frac{d^2 z}{d t^2} + b \frac{dz}{dt} + c \cdot z = F_w + F_k = F_k \]

Calculating diffraction force

\[(m + a) \frac{d^2 z}{d t^2} + b \frac{dz}{dt} + c \cdot z = F_W + F_k \]

Pressure / force due to undisturbed incoming wave

\[ F = - \int_s (p \cdot \vec{n}) dS \]

Pressure / force due to undisturbed incoming wave

\[ P = - \frac{\partial \Phi}{\partial t} \]

left hand side: reaction forces

\[(m + a) \frac{d^2 z}{d t^2} + b \frac{dz}{dt} + c \cdot z = F_W = F_k + F_D \]
Hydrodynamic coefficients

Determination of a and b:
- Forced oscillation with known frequency and amplitude
- Measure force needed to oscillate the model

For each of the 6 possible motions, the flow is described by a radiation potential function. The incoming waves are ignored for this. By finding a description of the flow, the pressures and consequently the forces can be determined later.

\[ (m + a) \ddot{z} + b \dot{z} + c \cdot z = F_{oscillation} \]

\[ z = z_0 \cos \omega t - \omega b z \sin \omega t = -\omega b z \cos (\omega t) \]

\[ \omega (m + a) \ddot{z_0} + c \dot{z_0} \cdot z_0 = F_{oscillation} \cos (\omega t + \phi_{oscillation}) \]

\[ \omega^2 a z_0 \cos \omega t - \omega b \dot{z_0} \sin \omega t = F_{oscillation} \cos (\omega t + \phi_{oscillation}) \]

\[ \omega^2 m \dot{z_0} \cos \omega t = F_{oscillation} \cos (\omega t + \phi_{oscillation}) \]

\[ \omega^2 (m - c) \dot{z_0} \sin \omega t \cos \omega t = F_{oscillation} \cos (\omega t + \phi_{oscillation}) \]

\[ \omega^2 m \dot{z_0} \cos \omega t = F_{oscillation} \cos (\omega t + \phi_{oscillation}) \]

Determination added mass and damping

Experimental procedure:

- Oscillate model (i.e. impose known harmonic motion, \( \omega \))
- Measure required force, \( F_{oscillation} \)
- Subtract known reaction forces from measured \( F_{oscillation} \)
- Split remainder into damping and added mass coefficient

\[ (m + a) \ddot{z} + b \dot{z} + c \cdot z = F_{oscillation} \]

Calculating hydrodynamic coefficients

- Oscillation in desired direction in still water
- To prevent water from penetrating through the hull, we need the radiation velocity potentials: \( \Phi_1 - \Phi_2 \)
- From potentials, we can calculate forces on body and the corresponding coefficients

\[ \text{Heave: } \Phi_2 \]

\[ \text{Oscillation in still water} \]
The boundary conditions are the same as those used for the undisturbed wave (Ch 5), however, we have an additional boundary now which is the hull of the structure; it has to be water tight!

For the radiation potentials, this means the flow in normal direction to the hull has to equal the velocity of the hull in normal direction, at every location. For the diffraction and the disturbed wave potential it means that the normal velocity due to their sum must be zero (the hull has no velocity itself).

But we have to take care in not introducing a tangential velocity! This means that the normal velocity due to their sum must be zero (since the structure has no velocity itself).

### Equation of motion

\[(m + a) \ddot{z} + b \dot{z} + c \cdot z = F_w\]

To solve equation of motion for certain frequency:

- Determine spring coefficient:
  - \( c \rightarrow \) follows from geometry of vessel

- Determine required hydrodynamic coefficients for desired frequency:
  - \( a, b \) \( \rightarrow \) computer / experiment

- Determine amplitude and phase of \( F_w \) of regular wave with amplitude =1:
  - Computer / experiment: \( F_w = F_{w0}\cos(\omega t + \phi_{w0}) \)

- As we consider the response to a regular wave with frequency \( \omega \):
  - Assume steady state response: \( z = z_w \cos(\omega t + \phi_{zw}) \)
  - Substitute in equation of motion:

\[
(m + a) \ddot{z} + b \dot{z} + c \cdot z = F_w
\]

\[
z = z_w \cos(\omega t + \phi_{zw})
\]

Now solve the equation for the unknown motion amplitude \( z_w \) and phase angle \( \phi_{zw} \)
Equation of motion

\[(m + a) \cdot \ddot{z} + b \cdot \dot{z} + c \cdot z = F_W\]

Now solve the equation for the unknown motion amplitude \(z \_a\) and phase angle \(\varepsilon\) for 1 frequency.

System is linear

If wave amplitude doubles \(\rightarrow\) wave force doubles \(\rightarrow\) motion doubles

\[(m + a) \cdot \ddot{z} + b \cdot \dot{z} + c \cdot z = F_W\]

Substitute solution \(z = \zeta \_a \cos(\omega t + \varepsilon)\) and solve RAO and phase.

Calculated RAO spar with potential theory

Frequency Response of semi-submersible

RAO

\[(m + a) \cdot \ddot{z} + b \cdot \dot{z} + c \cdot z = F_W\]

Moors dominated by wave force

Moors dominated by damping terms

Moors dominated by mass terms

Frequency (rad/s)

0          1          2          3          4          5

0          1          2          3          4          5
Learning goals Module II, behavior of floating bodies in waves

1. Motion Response in regular waves:
   - Linear waves:
     - "nearly" regularly harmonic (cosine shaped) waves
     - Wave steepness small: free surface boundary condition satisfied at mean still water level
     - Motion response in regular waves from RAO's and wave spectrum

2. Linearized wave exciting forces:
   - Wave force independent of motions
   - Wave force on mean wetted surface

3. Motion amplitudes are small:
   - Restoring force proportional to motion amplitude
   - Hydrodynamic reaction force proportional to motion amplitude

Learning goals Module II, behavior of floating bodies in waves

1. Motion Response in irregular waves:
   - Linearized wave exciting force:
     - Wave force independent of motions
     - Wave force on mean wetted surface
   - Restoring force proportional to motion amplitude
   - Hydrodynamic reaction force proportional to motion amplitude

2. Motion Response in irregular waves:
   - Wave force only on mean wetted surface
   - Motion response in irregular waves from RAO's and wave spectrum

3. Motion Response in irregular waves:
   - Motion response in irregular waves from RAO's and wave spectrum without forward speed

4. Nonlinear behavior:
   - Calculate average horizontal wave force on wall
   - Use of long-term motion equation

What is ‘linear’???

1. Linear waves:
   - "nearly" regularly harmonic (cosine shaped) waves
   - Wave steepness small: free surface boundary condition satisfied at mean still water level
   - Motion response in regular waves from RAO's and wave spectrum

2. Linearized wave exciting forces:
   - Wave force independent of motions
   - Wave force on mean wetted surface

3. Motion amplitudes are small:
   - Restoring force proportional to motion amplitude
   - Hydrodynamic reaction force proportional to motion amplitude

2D Potential theory (strip theory) p. 7-12 until p. 7-35

Structural aspects:
- Calculate structural forces and bending moments due to waves
- Nonlinear behavior:
  - Calculate average horizontal wave force on wall
  - Use of long-term motion equation

Today

Motion Response in regular waves:
- Linear waves:
  - "nearly" regularly harmonic (cosine shaped) waves
  - Wave steepness small: free surface boundary condition satisfied at mean still water level
  - Motion response in regular waves from RAO's and wave spectrum

Motion Response in irregular waves:
- Linearized wave exciting force:
  - Wave force independent of motions
  - Wave force on mean wetted surface
- Restoring force proportional to motion amplitude
- Hydrodynamic reaction force proportional to motion amplitude

Motion Response in irregular waves:
- Motion response in irregular waves from RAO's and wave spectrum
- Motion response in irregular waves from RAO's and wave spectrum without forward speed
- Motion Response in irregular waves:
  - Wave force only on mean wetted surface
  - Motion response in irregular waves from RAO's and wave spectrum

Nonlinear behavior:
- Calculate average horizontal wave force on wall
- Use of long-term motion equation

Structural aspects:
- Calculate structural forces and bending moments due to waves
- Nonlinear behavior:
  - Calculate average horizontal wave force on wall
  - Use of long-term motion equation
Calculating hydrodynamic coefficients and diffraction force

\[(m + a)z + b \cdot z + c \cdot z = F_w = F_{\alpha x} + F_{\alpha y}
\]

\[m \cdot \ddot{z} = \sum F_j = F_{\alpha x} + F_{\alpha y} + F_{\alpha z}
\]

\[\text{Radiation force: } F_{\alpha z} = -a_z \cdot z \cdot b \cdot z
\]

To calculate force: first describe fluid motions due to given heave motion by means of radiation potential:

\[F = -\int \left( \rho \cdot \nabla \right) dS
\]

\[M = -\int \rho \cdot (\tau \times \pi) dS
\]

\[p = -\frac{\rho}{i} \frac{\partial \Phi}{\partial t}
\]
Potential theory

Radiation potential

\( (m + \alpha) z + b \cdot \dot{z} + c \cdot \ddot{z} = F_x + F_z \)

Radiation potential heave \( \Phi_1(x, y, z, t) \)

Flow due to motions, larger motions \( \rightarrow \) 'more' flow

Problem: But we don't know the motions!! (we need the flow to calculate the motions...and we need the motions to calculate the flow...)

Solution: radiation potential is written as function of motion:

\[
\Phi_1(x, t) = \Re \{ \Phi_1(x, \xi(t)) \}
\]

Only space dependent

Only time dependent

Potential not necessarily in phase with heave velocity \( v_\xi \rightarrow \)

\( \Phi_1(x) = \text{complex amplitude of heave radiation potential (divided by } -\iota \omega z_\xi) \)
Potential theory

Let's consider forces and moments due to heave motion

\[ F_{x3} = \oint_S \rho \frac{\partial \Phi}{\partial t} \cdot n \, dS \]

\[ M_{y3} = \oint_S \rho \frac{\partial \Phi}{\partial t} \cdot (\mathbf{r} \times \mathbf{n}) \, dS \]

\[ \Phi(x,t) = \Phi \cdot v_3 \cdot e^{-i \omega t} \]

\[ \omega = \omega_0 \cdot s_{x3} \cdot e^{-i \omega t} \]

Potential theory

Radiation force due to heave motion is 3 component vector:

\[ F_{x3} = \Re \left\{ -\rho \cdot \mathbf{w} \cdot \mathbf{s}_{x3} \oint_S \frac{\partial}{\partial t} \Phi \cdot n \, dS \cdot e^{i \omega t} \right\} \]

\[ F_{y3} = \Re \left\{ -\rho \cdot \mathbf{w} \cdot \mathbf{s}_{y3} \oint_S \frac{\partial}{\partial t} \Phi \cdot n \, dS \cdot e^{i \omega t} \right\} \]

\[ F_{z3} = \Re \left\{ -\rho \cdot \mathbf{w} \cdot \mathbf{s}_{z3} \oint_S \frac{\partial}{\partial t} \Phi \cdot n \, dS \cdot e^{i \omega t} \right\} \]

In the following, only heave force due to heave motion is considered: \( F_{x3} \)
Potential theory

Radiation potential

\[ F_{R3} = \mathbb{R} \left\{ -\rho \cdot \omega^2 \cdot \phi \cdot s \cdot n \cdot dS \cdot e^{-i\omega t} \right\} \]

- Radiation force in heave direction, due to heave motion.

\[ m \cdot \ddot{z} + c \cdot \ddot{z} + F_{R3} = F_{a3} + F_{b3} \]

\[ \mathbb{R} \left\{ m \cdot \phi \cdot e^{i\omega t} + c \cdot \phi \cdot e^{i\omega t} \right\} = F_{a3} + F_{b3} \]

\[ \mathbb{R} \left\{ -m \cdot \phi \cdot e^{i\omega t} \right\} = F_{a3} + F_{b3} \]

- Heave force due to heave motion.

\[ F_i = \mathbb{R} \left\{ -\rho \cdot \omega^2 \cdot \phi \cdot s \cdot n \cdot dS \cdot e^{-i\omega t} \right\} \text{Heave force due to heave motion} \]

This force has a certain phase angle with respect to motion:

- Part in phase with motion acceleration is:

\[ \mathbb{R} \left\{ \frac{d^2}{dt^2} \cdot s \cdot e^{-i\omega t} \right\} = \mathbb{R} \left\{ \frac{d^2}{dt^2} \right\} \cdot s_{a3} = \mathbb{R}\left\{ \frac{d^2}{dt^2} \right\} \cdot a_{a3} \]

- Part in phase with motion velocity is:

\[ \mathbb{R} \left\{ \frac{d}{dt} \cdot s \cdot e^{-i\omega t} \right\} = \mathbb{R} \left\{ \frac{d}{dt} \right\} \cdot s_{a3} = \mathbb{R}\left\{ \frac{d}{dt} \right\} \cdot v_{a3} \]

\[ (m + \rho \cdot a_{a3}) \cdot \ddot{z} + 2\beta \cdot a_{a3} + \gamma = F_w \]
Potential theory

heave:

\[ \frac{\partial}{\partial t} \epsilon_{ij} = \gamma_{ij} \]

\[ \frac{\partial}{\partial t} (\epsilon_{ij} \epsilon^{mn}) - \varphi_{ij} \varphi_{ij} = 0 \]

Verify that

\[ \gamma_{ij} = -\varphi \left\{ \frac{\partial (\rho \cdot n)}{\partial x_i} \right\} \]

\[ \varphi_{ij} = -3 \left\{ \rho \frac{\partial (\rho \cdot n)}{\partial x_i} \right\} \]

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Potential theory

resulting from heave motions, \( \Phi \),

Forces

- \( a_k = -\varphi \left\{ \frac{\partial (\rho \cdot n)}{\partial x_k} \right\} \)
- \( b_k = -3 \left\{ \rho \frac{\partial (\rho \cdot n)}{\partial x_k} \right\} \)

Moments

- \( a_i = -\varphi \left\{ \frac{\partial (\rho \cdot n)}{\partial x_i} \right\} \)
- \( b_i = -3 \left\{ \rho \frac{\partial (\rho \cdot n)}{\partial x_i} \right\} \)

Solving the Laplace equation

coupled equation of motion:

\[
\begin{bmatrix}
    \Delta x_1 & \Delta x_2 & \Delta x_3 & \Delta x_4 & \Delta x_5 & \Delta x_6 \\
    \Delta y_1 & \Delta y_2 & \Delta y_3 & \Delta y_4 & \Delta y_5 & \Delta y_6 \\
    \Delta z_1 & \Delta z_2 & \Delta z_3 & \Delta z_4 & \Delta z_5 & \Delta z_6 \\
    \Delta x_1 & \Delta x_2 & \Delta x_3 & \Delta x_4 & \Delta x_5 & \Delta x_6 \\
    \Delta y_1 & \Delta y_2 & \Delta y_3 & \Delta y_4 & \Delta y_5 & \Delta y_6 \\
    \Delta z_1 & \Delta z_2 & \Delta z_3 & \Delta z_4 & \Delta z_5 & \Delta z_6 \\
    \Delta x_1 & \Delta x_2 & \Delta x_3 & \Delta x_4 & \Delta x_5 & \Delta x_6 \\
    \Delta y_1 & \Delta y_2 & \Delta y_3 & \Delta y_4 & \Delta y_5 & \Delta y_6 \\
    \Delta z_1 & \Delta z_2 & \Delta z_3 & \Delta z_4 & \Delta z_5 & \Delta z_6 \\
    \Delta x_1 & \Delta x_2 & \Delta x_3 & \Delta x_4 & \Delta x_5 & \Delta x_6 \\
\end{bmatrix}
\]
Sources images

[1] Towage of SSDR Transocean Amirante, source: Transocean
[4] Bluewater Rig No. 1, source: Friede & Goldman, LTD/GNU General Public License
[5] Source: unknown
[6] Rig Neptune, source: Seafarer Media
[7] Pieter Schelte vessel, source: Excalibur
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[13] Schematic of Curlew Field, United Kingdom, source: offshore-technology.com
[14] Ocean Quest Brave Sea, source: Zamakona Yards
[15] Medusa, A Floating SPAR Production Platform, source: Murphy USA