Hydrology of catchments, rivers and deltas (CIE5450)

Prof.dr.ir. Savenije

Lecture ‘Salinity and tides in alluvial estuaries’
Salinity and Tides in Alluvial Estuaries

by Hubert H.G. Savenije
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Salinity and Tides in Alluvial Estuaries

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“Salinity and Tides in Alluvial Estuaries” presents an integrated theory for one-dimensional flow and transport in estuaries with an active morphology. The book provides a new approach with numerous case illustrations and a comprehensive overview of the literature to date.

The author has many years of field experience in estuaries in Africa, Asia and Europe. He worked from 1978-1985 in Mozambique where he carried out a large number of surveys in four different estuaries and laid the foundation for the general theory presented in this book. Subsequently he worked as a consultant, mostly in Asia, where he verified the generality of the theory in estuaries in Thailand, Indonesia and Vietnam. He joined the UNESCO Institute for Water Education (IHE-Delft) in 1990, where he completed his PhD on the subject of this book. He became Professor of Water Resources Management at UNESCO-IHE in 1994 with a focus on river basin modelling and global water resources issues. He has been Professor of Hydrology at Delft University of Technology since 2000.

This book is a valuable theoretical resource for graduate students specialising in estuary processes and for researchers in related disciplines. It is also a useful guide for practitioners and consultants with its wide range of analytical equations, describing hydraulic, mixing and salt intrusion processes in estuaries.
Contents

• Estuary shape
• 5 new equations
• The role of the phase lag
• The role of tidal damping/amplification
• New versus “Classical” equations
Geometry of the Schelde estuary

\[ B = B_0 \exp\left(-\frac{x}{b}\right) \]

Geometry of the Incomati estuary

\[ A = h_0 B_0 \exp\left(-\frac{x}{b}\right) \]
<table>
<thead>
<tr>
<th>Equation Name</th>
<th>Newly derived equation</th>
<th>&quot;Classical&quot; equation</th>
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<tbody>
<tr>
<td>Phase Lag equation</td>
<td>( \tan \varepsilon = \frac{\omega b}{c(1 - \delta b)} = \frac{b}{\lambda (1 - \delta b)} )</td>
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<td>Geometry-Tide relation</td>
<td>( \frac{H}{E} = \frac{\eta \omega}{\nu} = \frac{\bar{h}}{r_s b} \frac{(1 - \delta b)}{\cos(\varepsilon)} )</td>
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<td>Scaling equation</td>
<td>( r_s \frac{\eta}{h} = \frac{\nu}{c} ) ( \frac{1}{\sin(\varepsilon)} )</td>
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<td>Damping equation Green’s eq. (1937)</td>
<td>( \frac{d H}{d x} \left(1 + \frac{g \eta}{\nu \sin \varepsilon}\right) = H\left(1 - \frac{f' \nu \sin \varepsilon}{\bar{h} c}\right) )</td>
<td>( \frac{d H}{d x} = H \left(1 - \frac{1}{2b}\right) )</td>
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<td>Celerity equation</td>
<td>( c^2 = \frac{1}{r_s} \frac{gh}{\left[1 - \frac{\sin 2\varepsilon}{2(1 + \alpha)} \left(\frac{c}{\omega b} - R'\right)\right]} )</td>
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<td>Mazure's equation (1837)</td>
<td>( c = \frac{\omega gh}{f \nu^2} \sin \varepsilon )</td>
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Most important differences

• Inclusion of Phase lag $\varepsilon$ between HW and HWS
• Inclusion of tidal damping $\delta$
The Phase Lag between HW and HWS
Progressive wave $\varepsilon = \pi/2$

Mixed wave $0 < \varepsilon < \pi/2$

Standing wave $\varepsilon = 0$

$$U = -\nu \sin(\omega t - \varepsilon)$$

$$Z = \eta \cos(\omega t) + \bar{h}$$

Progressive wave $\varepsilon = \pi/2$
Phase Lag equation

Conditions at HW:
\[ \frac{dh}{dt}=0, \; V=\upsilon \sin \varepsilon, \; \frac{dV}{dt}=\omega \upsilon \cos \varepsilon \]

Subst. in Lagrangean Continuity Equation:

\[ \tan \varepsilon = \frac{\omega b}{c(1-\delta b)} = \frac{b}{\lambda (1-\delta b)} \frac{2\pi}{(1-\delta b)} \]

(Savenije, 1993)
Tidal Damping and Amplification
Tidal damping and amplification

Tidal range in the Schelde

\[ y = \frac{\eta}{\eta_0} \]

- **Q=41 calc**
- **Q=41 obs**
- **Q=0 calc**
- **Q=100 calc**
Tidal damping and amplification

Tidal range in the Incomati

\[ y = \frac{\eta}{\eta_0} \]

- **calc**
- **obs**
Tidal damping and amplification

1. Combination of continuity and momentum balance equation
2. Impose constraint for HW: \( \frac{\partial h}{\partial t} = 0 \) and \( V = \nu \sin \varepsilon \)

3. Impose constraint for LW: \( \frac{\partial h}{\partial t} = 0 \) and \( V = -\nu \sin \varepsilon \)
4. Subtract the two envelopes
5. Yields:

\[
\delta = \frac{1}{\eta} \frac{d \eta}{d x} = \left( \frac{1}{b} - f' \frac{\nu \sin \varepsilon}{hc} \right) \left( 1 + \frac{g \eta}{c \nu \sin \varepsilon} \right)
\]

(Savenije, 1998)
Tidal damping and amplification

\[ \delta = \frac{1}{y} \frac{d y}{d x} = \left( \frac{1}{b} - f' \frac{\nu \sin \varepsilon}{hc} \right) \left( \frac{\alpha}{\alpha + y} \right) \]

\[ \alpha = O(0.1) \]

If \( y > 0.5 \) : linear amplification of damping

If \( y < 0.1 \) : exponential damping
The geometry-tide relation

1. Lagrangean version of Continuity equation
2. Integration between LW and HW
3. Taylor series expansion for $E/b < 1$

$$\frac{H}{E} = \frac{\eta \omega}{\nu} = \frac{\bar{h}}{r_s b \cos(\varepsilon)} (1 - \delta b)$$

(Savenije, 1993)
The Scaling equation

Combination of the geometry-tide relation with the Phase-Lag equation yields the Scaling Equation

\[ r_s \frac{\eta}{h} = \frac{\nu}{c \sin(\varepsilon)} \]

(Savenije & Veling, 2005)
Relation between wave celerity and tidal damping
Tidal propagation in the Schelde

Classical equation

\[ c_0 = \sqrt{\frac{1}{\beta} gh} \]

\[ y = \frac{H}{H_0} = \frac{\eta}{\eta_0} \]
Tidal propagation in the Incomati

Classical equation

\[ c_0 = \sqrt{\frac{1}{\beta} gh} \]

Tidal range in the Incomati

\[ y = \frac{H}{H_0} \]
The Celerity equation

1. Allow amplification and damping of the tide
2. Assume that the scaled tidal wave propagates undeformed
3. Assume $\eta/h<1$
4. Method of Characteristics
5. Solution for HWS and LWS

$$c^2 = \frac{1}{r_S} \frac{gh}{1 - \sin 2\varepsilon \left( \frac{c}{\omega b} - \frac{R'}{\omega} \right)}$$

(Savenije & Veling, 2005)
Comparing the new Equations with their Classical Counterparts
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<td>$c = \frac{\omega gh \eta}{f \nu^2}$</td>
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Conclusions

• The new equations demonstrate substantial differences with “classical” counterparts
• They are more general versions of the classical equations (which are special cases)
• They are consistent with each other (which the classical equations are not)
• The phase lag plays a key role and is the most important estuary parameter
Thank You for your attention
Some useful estuary equations

\[ \beta \frac{\eta \omega}{\nu} = \frac{h}{b} \left(1 - \delta b\right) \cos \epsilon = \beta \frac{H}{E} \]
continuity equation (Savenije, 1993)

\[ \tan \epsilon = \frac{\omega b}{(1 - \delta b)c} \]
from continuity equation (Savenije, 1993)

\[ \beta \frac{\eta}{h} \sin \epsilon = \frac{\nu}{c} = F \]
“the Scaling Equation” (Savenije & Veling, 2005)
References:


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