Hydrological Measurements

Prof. Wim Bastiaanssen

6. Modelling Evaporation
Modelling Evaporation

Prof. Wim Bastiaanssen
ET for hydrological studies

Source unknown
“actual” ET is unpredictable

EB can ‘see’ impacts on ET caused by:

- water shortage
- disease
- crop variety
- planting density
- cropping dates
- salinity
- management
ET for environmental studies

Annual evap Year 2000

Source unknown
For solving international conflicts
For verification of water use

<table>
<thead>
<tr>
<th>Avg ETa per plot</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>501 - 750</td>
<td></td>
</tr>
<tr>
<td>751 - 900</td>
<td></td>
</tr>
<tr>
<td>901 - 1,050</td>
<td></td>
</tr>
<tr>
<td>1,051 - 1,200</td>
<td></td>
</tr>
<tr>
<td>1,201 - 1,350</td>
<td></td>
</tr>
</tbody>
</table>
ET is calculated as a “residual” of the energy balance.

\[ ET = R_n - G - H \]

The energy balance includes all major sources \( (R_n) \) and consumers \( (ET, G, H) \) of energy.

- \( R_n \): net radiation
- \( G \): soil heat flux
- \( H \): sensible heat flux

February 28, 2013
Temperature is a function of ET
Surface temperature is a reflection of soil moisture

Source unknown
### Latent heat of vaporization

**TABLE 1**
Conversion factors for evapotranspiration

<table>
<thead>
<tr>
<th></th>
<th>depth</th>
<th>volume per unit area</th>
<th>energy per unit area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mm day⁻¹</td>
<td>m³ ha⁻¹ day⁻¹</td>
<td>l s⁻¹ ha⁻¹</td>
</tr>
<tr>
<td>1 mm day⁻¹</td>
<td>1</td>
<td>10</td>
<td>0.116</td>
</tr>
<tr>
<td>1 m³ ha⁻¹ day⁻¹</td>
<td>0.1</td>
<td>1</td>
<td>0.012</td>
</tr>
<tr>
<td>1 l s⁻¹ ha⁻¹</td>
<td>8.640</td>
<td>86.40</td>
<td>1</td>
</tr>
<tr>
<td>1 MJ m⁻² day⁻¹</td>
<td>0.408</td>
<td>4.082</td>
<td>0.047</td>
</tr>
</tbody>
</table>

* For water with a density of 1 000 kg m⁻³ and at 20°C.

**EXAMPLE 1**
Converting evaporation from one unit to another

On a summer day, net solar energy received at a lake reaches 15 MJ per square metre per day. If 80% of the energy is used to vaporize water, how large could the depth of evaporation be?

From Table 1:

<table>
<thead>
<tr>
<th></th>
<th>1 MJ m⁻² day⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Therefore:</td>
<td></td>
</tr>
<tr>
<td>0.8 x 15 MJ m⁻² day⁻¹ = 0.8 x 15 x 0.408 mm d⁻¹ =</td>
<td>0.408 mm day⁻¹</td>
</tr>
</tbody>
</table>

The evaporation rate could be 4.9 mm/day
Daily energy balance

FIGURE 5
Schematic presentation of the diurnal variation of the components of the energy balance above a well-watered transpiring surface on a cloudless day.
Radiation principles

Sun (5800K)
Scaled for Earth-Sun distance
Earth (288K)
**Planck equation, details**

Planck’s equation (the spectral curves shown)

\[ L_\lambda = \frac{2hc^2}{\lambda^5 \left(e^x - 1\right)}, \text{ where } x = \frac{hc}{k\lambda T} \]

Stefan-Boltzmann equation

\[ E = \pi \int L_\lambda d\lambda = \sigma T^4 \]

Wien’s displacement equation

\[ \lambda_{\text{max}} \left(\mu\text{m}\right) = \frac{2897}{T} \]

- \( c \) speed of light \( 3.00 \times 10^8 \) ms\(^{-1}\)
- \( h \) Planck’s constant \( 6.63 \times 10^{-34} \) Js
- \( k \) Boltzmann’s constant \( 1.38 \times 10^{-23} \) JK\(^{-1}\)
- \( \sigma \) Stefan-Boltzmann constant \( 5.67 \times 10^{-8} \) Wm\(^{-2}\)K\(^{-4}\)
- \( L_\lambda \) Spectral radiance Wm\(^{-2}\)m\(^{-1}\)sr\(^{-1}\)
Surface Radiation Balance

**Shortwave Radiation**

\[ R_S \downarrow \]
\[ \alpha R_S \]

**Longwave Radiation**

\[ R_L \downarrow \]
\[ (1-\varepsilon_o)R_L \downarrow \]
\[ R_L \uparrow \]

Vegetation Surface

Net Surface Radiation = **Gains – Losses**

\[ R_n = (1-\alpha)R_S \downarrow + R_L \downarrow - R_L \uparrow - (1-\varepsilon_o)R_L \downarrow \]
Net longwave radiation (1)

\[ R_{nl} = \sigma \left[ \frac{T_{\text{max},K}^4 + T_{\text{min},K}^4}{2} \right] \left( 0.34 - 0.14 \sqrt{e_a} \right) \left( 1.35 \frac{R_s}{R_{so}} - 0.35 \right) \]  \hspace{1cm} (39)

where:
- \( R_{nl} \) is net outgoing longwave radiation [MJ m\(^{-2}\) day\(^{-1}\)],
- \( \sigma \) is the Stefan-Boltzmann constant [4.903 \times 10^{-9} \text{ MJ K}^{-4} \text{ m}^{-2} \text{ day}^{-1}],
- \( T_{\text{max},K} \) is the maximum absolute temperature during the 24-hour period [K = °C + 273.16],
- \( T_{\text{min},K} \) is the minimum absolute temperature during the 24-hour period [K = °C + 273.16],
- \( e_a \) is the actual vapour pressure [kPa],
- \( R_s/R_{so} \) is the relative shortwave radiation (limited to \( \leq 1.0 \)),
- \( R_s \) is measured or calculated (Equation 35) solar radiation [MJ m\(^{-2}\) day\(^{-1}\)],
- \( R_{so} \) is calculated (Equation 36 or 37) clear-sky radiation [MJ m\(^{-2}\) day\(^{-1}\)].
**EXAMPLE 11**
**Determination of net longwave radiation**

In Rio de Janeiro (Brazil) at a latitude of 22°54'S (= -22.70°), 220 hours of bright sunshine, a mean monthly daily maximum and minimum air temperature of 25.1 and 19.1°C and a vapour pressure of 2.1 kPa were recorded in May. Determine the net longwave radiation.

<table>
<thead>
<tr>
<th>From Example 10:</th>
<th>From Eq. 36:</th>
<th>From Table 2.8 or for:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_\text{s}$ =</td>
<td>$R_{\text{so}} = 0.75 \ R_\text{a} = 0.75 \times 25.1 =$</td>
<td>$\sigma = 4.903 \times 10^{-9}$</td>
</tr>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$= 14.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_{\text{so}} = 18.8$</td>
<td></td>
</tr>
<tr>
<td>From Table 2.8 or for:</td>
<td>Then:</td>
<td></td>
</tr>
<tr>
<td>and:</td>
<td>$\sigma = 4.903 \times 10^{-9}$</td>
<td></td>
</tr>
<tr>
<td>$T_{\text{max}} = 25.1$°C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and:</td>
<td>$T_{\text{min}} = 19.1$°C</td>
<td></td>
</tr>
<tr>
<td>and:</td>
<td>$\sigma \ T_{\text{max}} \ K^4 = 298.3$</td>
<td></td>
</tr>
<tr>
<td>and:</td>
<td>$\sigma \ T_{\text{min}} \ K^4 = 35.8$ MJ m$^{-2}$ day$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>and:</td>
<td>$e_\text{a} = 0.34 - 0.14 \ \sqrt{e_\text{a}} =$</td>
<td></td>
</tr>
<tr>
<td>and:</td>
<td>$0.34 - 0.14 \ \sqrt{0.408} =$</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>$= 3.5$ MJ m$^{-2}$ day$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>$= 4.903 \times 10^{-9}$</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>$= 3.5$ MJ m$^{-2}$ day$^{-1}$</td>
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<tr>
<td>-</td>
<td>$= 3.5$ MJ m$^{-2}$ day$^{-1}$</td>
<td></td>
</tr>
</tbody>
</table>

The net longwave radiation is 3.5 MJ m$^{-2}$ day$^{-1}$. 

**Net longwave radiation (2)**
Logarithmic wind profile

Wind Profiles

$T_{\text{air}} - T_{\text{water}} = -4$
$T_{\text{air}} - T_{\text{water}} = -2$
$T_{\text{air}} - T_{\text{water}} = 0$
$T_{\text{air}} - T_{\text{water}} = 2$
$T_{\text{air}} - T_{\text{water}} = 4$

$U(z=6) = 6 \text{ m/s}$
$q_{\text{stc}} - q(z=6) = 3 \text{ g/kg}$
Effect of buoyancy on turbulent transport

Neutral = no convection
Stable = heat towards land
Unstable = heat away from land

$U(z)$ wind speed
Vertical wind profile – neutral conditions

\[ u_* = \frac{k \cdot u_{z1}}{\ln \left( \frac{z_1}{z_{om}} \right)} \]
Vertical wind profile – non neutral conditions

\[ u_* = \frac{k \cdot u_z}{\ln \left( \frac{z_1}{z_{om}} \right) - \psi_m \left( \frac{z_1}{L} \right) + \psi_m \left( \frac{z_{om}}{L} \right)} \]

\[ L = \frac{-\rho_a \cdot c_p \cdot T \cdot u_*^3}{k \cdot g \cdot H} \]

Monin Obukhov Length

\[ \psi_m \left( \frac{z}{L} \right) = \begin{cases} 
L < 0 : 2 \cdot \ln \left( \frac{1 + x}{2} \right) + \ln \left( \frac{1 + x^2}{2} \right) - 2 \cdot \arctan(x) + \frac{\pi}{2} \\
L = 0 : 0 \\
L > 0 : -5 \cdot \frac{z}{L} 
\end{cases} \]

\[ x = 4 \sqrt{1 - 16 \cdot \frac{z}{L}} \]
Flux – profile relationships for momentum, heat and vapor

\[
\begin{align*}
T_{cb} & \quad u_{cb} & \quad T_{cb} & \quad u_{cb} \\
T^*{(1)} & \quad u^*{(1)} & \quad T^*{(2)} & \quad u^*{(2)} \\
T_{\text{meas}} & \quad u_{\text{meas}} & \quad T_{\text{calc}} & \quad u_{\text{calc}}
\end{align*}
\]
Wind and temperature vertical profiles

\[ u_{z2} = u_{z1} + \frac{u_*}{k} \left\{ \ln \left( \frac{z_1}{z_2} \right) - \psi_m \left( \frac{z_1}{L} \right) + \psi_m \left( \frac{z_2}{L} \right) \right\} \]

\[ T_{z2} = T_{z1} + \frac{T_*}{k} \left\{ \ln \left( \frac{z_1}{z_2} \right) - \psi_h \left( \frac{z_1}{L} \right) + \psi_h \left( \frac{z_2}{L} \right) \right\} \]

\[ \psi_h \left( \frac{z}{L} \right) = \begin{cases} L < 0 : & 2 \cdot \ln \left( \frac{1}{2} + \sqrt{\frac{1}{4} - 4 \cdot \frac{z}{L}} \right) \\ L = 0 : & 0 \\ L > 0 : & -5 \cdot \frac{z}{L} \end{cases} \]
Heat flux and scalars

\[ T_* = \frac{-H}{\rho_a \cdot c_p \cdot u_*} \]
Sensible Heat Flux (H) written as Ohm’s law

\[ H = \left( \rho \times c_p \times \frac{dT}{r_{ah}} \right) \]

\( dT = \) the near surface temperature difference (K).

\( r_{ah} = \) the aerodynamic resistance to heat transport (s/m).

Neutral conditions

\[ r_{ah} = \frac{\ln\left( \frac{z_2}{z_1} \right)}{u^* \times k} \]

\( u^* = \) friction velocity [m/s]
Stability correction for buoyancy

\[ u^* = \frac{u_{200}k}{\ln \left( \frac{200}{z_{0m}} \right) - \Psi_{m(200m)} + \Psi_{m(z0m)}} \]

\[ r_{ah} = \frac{\ln \left( \frac{z_2}{z_1} \right) - \Psi_{h(z_2)} + \Psi_{h(z_1)}}{u^* \times k} \]
Transfer equation sensible heat

\[ H = -\rho c_p u_* T_* = \rho c_p C_h U (T_0 - T_a) = \rho c_p \left[ \frac{(T_0 - T_a)}{r_{ah}} \right], \]

(3)
Soil heat flux

\[ J_q = -K_q \frac{\partial T}{\partial x} \]
Transpiration process
Soil evaporation process

Source unknown
Transfer equation for latent heat

\[ LE = \lambda \rho_{air} C_E u (q_{satTs} - q_a) , \]

where

- \( \rho_{air} \) is the density of moist air, \( \text{kg/m}^3 \),
- \( C_E \) is a bulk transfer coefficient for water vapor, dimensionless,
- \( u \) is wind speed, in m/s,
- \( q_{satTs} \) is saturated specific humidity at surface temperature, in kg/kg,
- \( q_a \) is specific humidity at observation height, kg/kg.
Slope of the saturated vapor pressure curve

\[ E_{\text{sat}}(T_0) = e_{\text{sat}}(T_a) + \text{SLOPE} \ (T_0 - T_a) \]

Slope = \Delta

Source unknown
Penman – Monteith equation

\[ \Delta (R_n - G) + \rho \ c_p \ \text{vpd} / r_a \]

LE = \[ \Delta + \gamma \ (1 + r_s / r_a) \]

Bio-physical parameters (besides weather parameters)

- Albedo
- Emissivity
- G/R_n
- Surface roughness, r_a
- Stomatal resistance, r_s
- LAI, r_s
Canopy resistance model:

\[ r_c = \frac{r_{\text{sm}}}{\text{LAI}} \{\phi_{\text{par}} \phi_{\text{temp}} \phi_{\text{vpd}} \phi_{\text{mois}}\} \]
Soil moisture and surface resistance

\[ \frac{1}{r_c} = f_1(T) f_2(\text{VPD}) f_3(\text{PAR}) f_4(\psi) / r_{cMIN} \]

- Temperature
- Solar radiation
- Humidity
- Soil water

This solves the ET from MICW measurements.
FIGURE 9
Characteristics of the hypothetical reference crop

\[ r_a = \frac{208}{u_2} \text{ s/m} \]

\[ \alpha R_s = 0.23 R_s \]

\[ r_s = 70 \text{ s/m} \]

\[ h = 0.12 \text{ m} \]

Source unknown
The Penman-Monteith form of the combination equation is:

\[
\lambda ET = \frac{\Delta (R_n - G) + \rho_a c_p (e_s - e_a)}{r_a} \frac{r_s}{r_a} \\
\Delta + \gamma \left(1 + \frac{r_s}{r_a}\right)
\]  

(3)
Aerodynamic resistance

BOX 4
The aerodynamic resistance for a grass reference surface

For a wide range of crops the zero plane displacement height, \( d \) [m], and the roughness length governing momentum transfer, \( z_{om} \) [m], can be estimated from the crop height \( h \) [m] by the following equations:

\[
    d = \frac{2}{3} h \\
    z_{om} = 0.123 h
\]

The roughness length governing transfer of heat and vapour, \( z_{oh} \) [m], can be approximated by:

\[
    z_{oh} = 0.1 \ z_{om}
\]

Assuming a constant crop height of 0.12 m and a standardized height for wind speed, temperature and humidity at 2 m (\( z_m = z_h = 2 \ m \)), the aerodynamic resistance \( r_a \) [s m\(^{-1}\)] for the grass reference surface becomes (Eq. 4):

\[
    r_a = \frac{\ln \left( \frac{2-2/3(0.12)}{0.123(0.12)} \right) \ln \left( \frac{2-2/3(0.12)}{(0.1)0.123(0.12)} \right)}{(0.41)^2 u_2} = \frac{208}{u_2}
\]

where \( u_2 \) is the wind speed [m s\(^{-1}\)] at 2 m.
Potential ET for correction of grass

Figure 20
Typical $K_c$ for different types of full grown crops

- Pineapple
- Citrus
- Cherries
- Peaches
- Cotton
- Large Vegetables
- Small Vegetables
- Maize
- Sugar Cane

Source unknown
Crop coefficient

FIGURE 22
The effect of evaporation on $K_c$. The horizontal line represents $K_c$ when the soil surface is kept continuously wet. The curved line corresponds to $K_c$ when the soil surface is kept dry but the crop receives sufficient water to sustain full transpiration.