Problem 1.
Correct answer is D

The term snowball-effect describes the effect that if for one system (not only structural - B) the weight is reduced, this will have an impact on a number (not all - C) other systems as well (resulting in additional weight reductions). How the achieved weight reduction is used is not defined by the term (A).

Problem 2

a. For aluminum: Force = Allowable stress x width x thickness
   So, \( t = \frac{F}{\sigma w} \)
   In direction 1: \( t = \frac{75000}{100 \times 300} = 2.5 \text{ mm} \)
   In direction 2: \( t = \frac{120000}{100 \times 600} = 2 \text{ mm} \)
   The minimum required thickness is therefore 2.5 mm

   For Carbon composite we can use the same formula:
   In direction 1: \( t = \frac{75000}{150 \times 300} = 1.667 \text{ mm} \)
   In direction 2: \( t = \frac{120000}{150 \times 600} = 1.333 \text{ mm} \)
   Since the composite is UD, it is able to carry loads in ONE direction only. Therefore, we need to add the thicknesses.
   So, the minimum required thickness is this case is 3.0 mm

b. Weight = Width x Length x thickness x density = \( w \times L \times t \times \rho \)
   For aluminium: \( 0.3 \times 0.6 \times 0.0025 \times 2800 = 1.26 \text{ kg} \)
   For composite: \( 0.3 \times 0.6 \times 0.003 \times 1800 = 0.972 \text{ kg} \)

c. In this answer the choice is not important but the motivation!
   E.g. You can choose for aluminum with the argument that for small product series production costs are very important.
   You could also choose for composites with the argument that over the long life time of the product (30 years or more) the weight advantage of composite will result in much higher cost savings (less fuel required).
Problem 3

3a. [6 pts] Use standard atmosphere formulae for troposphere:
Calculate $T$ with linear gradient $a = -0.0065 \text{ K/m}$
$T = 216.65 \text{ K}, \ p = 22631.7 \text{ Pa}, \ \rho = 0.3639158 \text{ kg/m}^3$

3b. [5 pts] $L=W=2451662.5 \text{ N}$
$L = 0.432424350513$
$C_D = 0.0340695414415$
$T = D = 193159.837195 \text{ N} = 193 \text{ kN}$
$\text{Pa per eng} = 11924390.3143 \text{ W or 11924 kW}$

3c. [6 pts] Standard configuration, so see Stability & Control hand-out for derivation, also derive downwash derivative $d\rho_p/d\alpha$!

3d. [5 pts] From equation it follows: $l_{np} = 0.026607492312 \ S_h$
Using $c = 8.516667 \text{ m}$
$V_h = 0.544059$
$S_h = 75.1668 \text{ m}^2$

3e. [3 pts] Main advantage: - better longitudinal static stability, hence larger c.g. range
Two disadvantages: - more weight
- more drag
Problem 4

a) $M = \frac{V^2}{a}$

We know that $a = \sqrt{T/\rho}$ and $M = \frac{V}{a}$ by definition.

\[ V = \frac{810}{3.6} = 225 \text{ m/s} \]
\[ a = \sqrt{1.4 \times 207.05 \times 255.7} = 320.56 \text{ m/s} \]

\[ M = \frac{225}{320.56} \approx 0.706 \]

b) The profile lift gradient is defined as $\frac{\Delta C_l}{\Delta \alpha}$

\[ a_e = \frac{0.66 - 0}{4 - (-2)} = 0.11 \text{ per degree} \]

for the wing we can derive $a = \frac{a_0}{1 + \frac{a_0 \times 57.3}{\pi A e}}$

\[ A = \frac{b^2}{S} = \frac{34.5^2}{149} = 7.99 \]
\[ \epsilon = 0.82 \]

Hence $a = \frac{0.11}{1 + \frac{0.11 \times 57.3}{\pi \times 7.99 \times 0.82}} \approx 0.084$
This is however at the low Mach number. The aircraft flies at \( M = 0.7 \).

Using the Prandtl–Glauert correction for lift \( (C_L = \frac{C_{\infty}}{\sqrt{1-M^2}}) \), we arrive at \( a = \frac{0.0492}{\sqrt{1-0.7^2}} \) = 0.1179.

The lift of the wing at 3° and \( M = 0.7 \) is:

\[ C_L = 5 \times 0.1179 = 0.59 \]

We also have \( C_D = C_{\infty} + \frac{a^2}{2 \pi Re} \)
hence \( C_D = 0.0062 + \frac{0.59^2}{2 \times 10^6} = 0.0149 \)

and so we arrive at \( \frac{C_L}{C_D} = \frac{0.59}{0.0149} = 39.86 \).

1. In stagnation point is?

You can use the energy equation:

\[ C_p T_1 + \frac{1}{2} V_1^2 = C_p T_0 + \frac{1}{2} V_0^2 \]

or use \( \frac{T_0}{T_1} = 1 + \frac{\gamma-1}{\gamma} M_1^2 \), but note that

in the latter case the velocity in stagnation is zero.

With the energy equation \( a \) is also at the stagnation point.

\( V_1 = 0 : \quad T_1 = \frac{C_p T_0 + \frac{1}{2} V_0^2}{C_p} = 280.8 \text{ K} \)
Using the second form of the isentropic relations:

\[ \frac{T_0}{T_1} = 1 + \frac{x - 1}{2} M_1^2 \]

In this case station 0 is the stagnation point (where \( v_0 = 0 \), which was the input to the derivation of this formula).

\[ \frac{T_0}{T_1} = 1 + \frac{1.4 - 1 \times 0.7}{2} = 1.098 \]

\[ T_0 = T_1 \times 1.098 = 255.7 \times 1.098 = 280.8 \text{ K} \]

By definition:

\[ C_p = \frac{P - P_0}{\rho_0} \text{ or } \frac{P - P_0}{\rho_0} \]

\[ P = C_p \rho_0 + P_0 \]

\[ \rho_0 = \frac{1}{2} \rho v_0^2 = \frac{1}{2} \times 0.736 \times 225 = 1.2863 \times 10^4 \]

\[ P_0 = 5.41 \times 10^4 \text{ N/m}^2 \]

\[ P = -1.23 \times 1.2863 \times 10^4 + 5.41 \times 10^4 = 3.01 \times 10^4 \text{ N/m}^2 \]
Answer to 5a:

Steady: \( \frac{dV}{dt} = 0 \); straight: \( \frac{dy}{dt} = 0 \); Assumption: small angle approximation \( \rightarrow \cos(\gamma) \approx 0 \)

Thus, the equations of motion simplify to:

\[
T - D - W \sin \gamma = 0
\]

\[
L = W
\]

Multiply the first equation with airspeed in order to introduce rate of climb in the equation.

\[
TV - DV - WV \sin \gamma = 0
\]

\[
\frac{P_a - P_T}{W} = V \sin \gamma = RC
\]

Maximum rate of climb is achieved when the excess power \((P_a - P_T)\) is minimal since we assume a constant aircraft weight. Maximum power available is given to be independent of airspeed. Thus, maximum rate of climb occurs when power required is minimum.

\[
P_T = DV
\]

\[
P_T = \frac{D}{L} V = \frac{D}{L} WV = \frac{C_D}{C_L} WV
\]

\[
L = W \rightarrow V = \sqrt{\frac{W}{S}} \frac{2}{C_L}
\]

\[
P_T = \frac{C_D}{C_L} WV = \frac{C_D}{C_L} W \sqrt{\frac{W}{S}} \frac{2}{\rho C_L} = \sqrt{\frac{W^3}{S} \frac{2}{\rho} C_L^2}
\]

Aircraft weight and air density are given so the ratio \(C_L^3 / C_D^2\) should be maximum. This is the case when the derivative of this ratio to \(C_L\) is equal to 0.

\[
C_L = \sqrt[3]{3C_{D_o} \pi A_e}
\]

(this equation should be derived – the derivation can be found in the lecture sheets)

Fill in the take-off configuration values:

\[
C_L = \sqrt[3]{3 \cdot 0.03 \cdot \pi \cdot 5} = 1.19
\]

The drag coefficient, drag, power required and rate of climb can now easily be calculated

\[
C_D = C_{D_o} + \frac{C_L^2}{\pi A_e} = 0.03 + \frac{1.19^2}{5 \pi} = 0.12
\]

\[
P_T = \sqrt{\frac{W^3}{S} \frac{2}{\rho} C_L^2} = \sqrt{\frac{8500^3}{S} \frac{2}{\rho} C_L^2} = \sqrt{9.84 \cdot 29.5^3} = 29.5 \text{ [kW]}
\]

\[
RC = \frac{P_a - P_T}{W} = \frac{115 - 29.5}{8.5} = 10.1 \text{ [m/s]}
\]

The corresponding airspeed: \( V = \sqrt{\frac{W}{S} \frac{2}{\rho C_L}} = \sqrt{\frac{8500}{9.84 \cdot 29.5} \frac{2}{1.19}} = 34.4 \text{ [m/s]} \)
Answer to 5b:
Maximum horizontal distance
Steady: \( dV/dt = 0 \); straight: \( d\gamma/dt = 0 \); Assumption: small angle approximation \( \rightarrow \cos(\gamma) \equiv 0 \) and finally no engine thrust: \( T=0 \)
Thus the equations of motion simplify to:

\[-D - W \sin \gamma = 0\]
\[L = W\]

We need to know the flight path angle to calculate the distance:

\[\sin \gamma = \frac{-D}{W}\]
\[\sin \gamma = \frac{-D}{L} = \frac{C_D}{C_L}\]
\[\gamma = \arcsin\left(\frac{C_D}{C_L}\right)\]

So the maximum distance is achieved when the ratio \( C_L/C_D \) is maximum. This is the case when the derivative of the ratio to \( C_L \) equals 0.

\[C_L = \sqrt{C_{D_0} \pi Ae} \quad (this \ equation \ should \ be \ derived – \ the \ derivation \ can \ be \ found \ in \ the \ lecture \ sheets)\]
Fill in the clean configuration values:
\[C_L = \sqrt{0.02 \cdot \pi \cdot 5.5} = 0.59\]
The drag coefficient can now be calculated
\[C_D = C_{D_0} + \frac{C_L^2}{\pi Ae} = 0.02 + \frac{0.59^2}{5\pi} = 0.04\]
Hence,
\[\gamma = \arcsin\left(\frac{-0.04}{0.59}\right) = -3.9 \text{ [deg]}\]
The distance that can be flown follows from basic trigonometry \((3000/\tan(3.9))\) and equals 44 km.

Answer to 5c:
Answer C is correct

Answer to 5d:
Answer B is correct