Introduction to Aerospace Engineering

Exams
Answers exam AE1101 November 2010

English:

Question 1

Answers

  a. Vertical force results in tension (A, D) and compression (B, C)
  b. Horizontal force results in tension (A) and three Zero Force Elements (B, C, D)
  c. A diagonal bar from point 2 to point 4 is possible; in case of the vertical force
     A remains in tension; D will be in compression and B and C will be ZFE. In
     case of the horizontal force, nothing will change
  d. A web plate when air- or liquid tightness is required; when even shear
     distribution is favorable, etc.

Points: a = 4; b = 4; c = 4; d = 3

Dutch:

Vraag 1

Antwoorden

  a. Vertikale kracht geeft trekkracht in A en D, en drukkracht in B en C
  b. Horizontale kracht geeft trekkkracht in A en geen krachten in B, C en D
  c. Een diagonale staaf van punt 2 naar punt 4 is mogelijk; bij de vertikale kracht
     wordt A op trek belast, D op druk en B en C zijn onbelast; bij de horizontale
     kracht vindt er geen verandering plaats.
  d. Een lijfplaat kan gebruikt worden als lucht- of vloeistofdichtheid een vereiste
     is, als een gelijkmatige schuifbelasting beter is, enz.

Punten: a = 4; b = 4; c = 4; d = 3
Opgave 2
Points: \(a=4\) \(b=3\) \(c=3\) \(d=4\) \(e=3\) \(f=3\)

2a) ISA unfolding gradient wit shear:

\[
\frac{\rho_i}{\rho_0} = \left(\frac{T_i}{T_0}\right)^{-\frac{g}{c_\text{R}}} - 1
\]

\[\rho_0 \to T_0 = T_0 + \alpha h\]

\[
\rho_i = \rho_0 \left(\frac{T_0 + \alpha h}{T_0}\right)^{-\frac{g}{c_\text{R}}} - 1 = \rho_0 \left(1 + \frac{\alpha}{c_\text{R}} h\right)^{-\frac{g}{c_\text{R}}} - 1
\]

\[
\rho_i(h) = 1.225 \left(1 - \frac{h}{4431}\right)^{4.2577} \text{ h in [m]}
\]

\[h < 11000\]

b) 

\[
C_{\text{L,max}} = \frac{m \cdot g}{\frac{1}{2} \rho V_{\text{EAS}}^2 S}
\]

\(V_{\text{EAS}} \Rightarrow \rho = \rho_0\)

no flaps: \(C_{\text{L,max}} = 1.102 \approx 1.1\)

full flaps: \(C_{\text{L,max}} = 1.751 \approx 1.7\)

c) \(\rho(T_0E) = 0.3794 \text{ kg/m}^3 \quad (\text{w/c} 2\text{a})\)

\[V_{\text{TAS}} = \sqrt{\frac{\rho}{\rho}} V_{\text{EAS}} = \sqrt{\frac{1.225}{0.3794}} \cdot 94 = 169 \text{ kts} = 92 \text{ m/s}\]

d) \(\rho = \rho RT \Rightarrow \rho = \frac{p}{RT} = \frac{103000}{287.05 \cdot 55315} = 1.4174 \text{ kg/m}^3\)

\[V = \frac{W}{C_{\text{L,max}} \cdot \frac{1}{2} \rho S} \Rightarrow V_{\text{null no flaps}} = 44.95 \text{ m/s} = 87 \text{ kts}\]

\[V_{\text{null full flaps}} = 35.06 \text{ m/s} = 70 \text{ kts}\]
e) Lift is proportional to dynamic pressure $\frac{1}{2} \rho V^2$, so when density decreases ($\rho$)
the speed needs to increase to create sufficient lift. When it is colder, the air is thicker, so the stall speed can be less.

or: via formula show, $V_{stall} \sim \sqrt{\frac{1}{\rho}}$

2f

- Altitude limit – pressurized cabin (maximum pressure differential on fuselage structure)
- Design diving speed $V_D$ / Maximum operating speed $V_{MO}$. The airplane is designed to withstand particular flight loads at this speed. (Positive and negative gusts of 25 ft/s should be considered $V_D$)
- Design diving Mach number $M_D$ / Maximum operating Mach number $M_{MO}$. The aircraft is designed to remain controllable up to this speed. (undesirable flying qualities, associated with buffeting effect can occur above this Mach number).

(Note: the difference between $V_D$ and $V_{MO}$, or $M_D$ and $M_{MO}$, is a safety margin. MO stands for maximum operating)

3a
Free body diagram

Kinetic diagram
3b
\[ \sum F_{\parallel V} : \frac{W}{g} \frac{dV}{dt} = T \cos \alpha_t - D - W \sin \gamma \]
\[ \sum F_{\perp V} : \frac{W}{g} \frac{d\gamma}{dt} = L - W \cos \gamma + T \sin \alpha_t \]

3c
Horizontal flight: The aircraft remains at a constant altitude (\( \gamma = 0; \frac{d\gamma}{dt} = 0 \))
Steady flight: Flight in which the forces and moments acting on the aircraft do not vary in time, neither in magnitude, nor in direction (\( \frac{dV}{dt} = 0 \))
Symmetric flight: flight in which both the angle of sideslip is zero and the plane of symmetry of the aircraft is perpendicular to the earth (\( \beta = 0 \) and the aircraft is not turning)

\[ \frac{dV}{dt} = 0; \quad \gamma = 0; \quad \frac{d\gamma}{dt} = 0; \quad \alpha_t = 0 \]
\[ \sum F_{\parallel V} : \frac{W}{g} \cdot 0 = T \cos 0 - D - W \sin 0 \]
\[ \sum F_{\perp V} : \frac{W}{g} \cdot 0 = L - W \cos 0 + T \sin 0 \]
\[ L = W \]
\[ T = D \]

3d
The ratio \( \frac{V}{F} \) represents airspeed [m/s] divided by fuel flow [N/s] or [kg/s]. Hence:
\[
\begin{bmatrix} V \\ F \end{bmatrix} = \begin{bmatrix} \frac{m}{s} \\ \frac{N}{s} \end{bmatrix} = \begin{bmatrix} m \\ N \end{bmatrix}
\]

In other words, it is the distance that can be flown per unit of fuel. Clearly this must be maximized to obtain the maximum range.

3e
\[ F \triangleq c_pP_{br} \iff F = c_p \frac{P_s}{\eta} \]
\[ T = D \]
\[ P_s = P_r \]
$F = c_p \frac{P}{\eta_j} = c_p \frac{DV}{\eta_j}$

$V = \frac{\eta_j}{c_p} \frac{1}{D}$

$\eta_j$ and $c_p$ can be assumed to be constant as a function of airspeed (over the range of cruising speeds) for propeller aircraft

$R_{\text{max}} \Rightarrow \left(\frac{V}{F}\right)_{\text{max}} \Rightarrow D_{\text{min}} = \left(\frac{C_D}{C_L}\right)_{\text{min}} \Rightarrow \left(\frac{C_L}{C_D}\right)_{\text{max}}$

\[3f\]

\[
\frac{d}{dC_L} \left( \frac{C_L}{C_D} \right) = 0
\]

\[
C_D \cdot 1 - C_L \cdot \frac{dC_D}{dC_L} = 0
\]

\[
\frac{dC_D}{dC_L} = \frac{C_D}{C_L}
\]

\[
C_D = C_{D_0} + kC_L^2
\]

\[
2kC_L = \frac{C_{D_0} + kC_L^2}{C_L}
\]

\[
C_L = \sqrt{\frac{C_{D_0}}{k}} = \sqrt{\frac{0.0275}{0.0456}} = 0.78
\]

\[3g\]

$L=W$

\[
V = \sqrt{\frac{W}{S \cdot \rho \cdot C_L}} = \sqrt{\frac{45000 \cdot 2 \cdot 1}{28.8 \cdot 1.225 \cdot 0.78}} = 57.2 \text{ [m/s]}
\]

\[3h\]

\[
C_D = 0.0275 + 0.0456C_L^2 = 0.0275 + 0.0456 \cdot 0.78^2 = 0.055
\]

\[
\frac{V}{F} \cdot \frac{\eta_j}{c_p} \frac{1}{D} \cdot \frac{\eta_j}{c_p} \frac{C_L}{C_{D_W}} = \frac{0.35 \cdot 0.78 \cdot 1}{0.108 \cdot 10^{-6} \cdot 9.81 \cdot 0.055 \cdot 45000} = 104 \text{ [m/N]}
\]

\[3i\]

(The numbers in the figure are not correct, it is only intended to show qualitatively the shape of the figure)
Problem 4

The following data are given:

\[ T_0 = 3000 \text{ K} \quad A_t = 0.08 \text{ m}^2 \]
\[ p_0 = 15 \text{ atm} \quad p_e = 1 \text{ atm} \]
\[ R = 378 \frac{\text{ J}}{\text{ kg K}} \quad \gamma = 1.26 \]

a)

Since the mach number at the exit is given by \( M_e = \frac{V_e}{a_e} \) the speed at the exit is known when \( M_2 \) and \( a_2 \) are available. With \( a_e = \sqrt{\gamma R T_e} \) becomes clear that we have to calculate the exit temperature first.

Apply the isentropic relations:

\[ \frac{p_e}{p_0} = \left( \frac{T_e}{T_0} \right)^{\frac{\gamma}{\gamma-1}} \quad \text{With this relation we find the temperature in the exit:} \]

\[ T_e = T_0 \left( \frac{p_e}{p_0} \right)^{\frac{1}{\gamma-1}} \quad \Rightarrow \quad T_e = 1715.7 \text{ K} \]
\[ a_e = \sqrt{\gamma R T_e} \quad \Rightarrow \quad a_e = 903.96 \frac{\text{ m}}{\text{s}} \]

To find the mach number at the exit we apply the total pressure equation:

\[ \frac{p_0}{p_e} = \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{\frac{\gamma}{\gamma-1}} \quad \text{or:} \quad M_e = \sqrt{\left( \frac{p_0}{p_e} \right)^{\frac{\gamma}{\gamma-1}} - 1} \quad \Rightarrow \quad M_e = 2.4 \]

Hence the flow speed at exit becomes: \( V_e = M_e a_e \quad V_e = 2169.2 \frac{\text{ m}}{\text{s}} \)

b)

The mass flow is given by: \( m = \rho \cdot A \cdot V = \rho_f A_t V_t \) (index t = throat)

With supersonic speed in the exit the mach number at the throat is: \( M_t = 1 \)
The temperature in the throat is found by applying:

\[
\frac{T_0}{T_t} = 1 + \frac{\gamma - 1}{2} M_t^2 \quad \text{or} \quad T_t := \frac{T_0}{1 + \frac{\gamma - 1}{2} M_t^2} \quad T_t = 2654.9 \, \text{K}
\]

The speed of sound in the throat becomes:

\[a_t := \sqrt{\gamma R T_t} \quad \Rightarrow \quad a_t = 1124.5 \, \frac{\text{m}}{\text{s}}\]

and the flow speed:

\[V_t := M_t a_t \quad V_t = 1124.5 \, \frac{\text{m}}{\text{s}}\]

Finally the density in the throat is found from:

\[
\frac{\rho_t}{\rho_0} = \left(1 + \frac{\gamma - 1}{2} M_t^2\right)^{\frac{1}{\gamma - 1}}
\]

Where \(\rho_0\) is given by:

\[\rho_0 := \frac{p_0}{R T_0} \quad \Rightarrow \quad \rho_0 = 1.340 \, \frac{\text{kg}}{\text{m}^3}\]

Now:

\[\rho_t := \frac{\rho_0}{\left(1 + \frac{\gamma - 1}{2} M_t^2\right)^{\frac{1}{\gamma - 1}}} \quad \Rightarrow \quad \rho_t = 0.838 \, \frac{\text{kg}}{\text{m}^3}\]

And the massflow becomes:

\[\text{massflow} := \rho_t A_t V_t \quad \text{massflow} = 75.4 \, \frac{\text{kg}}{\text{s}}\]
Problem 4: continued

Alternatively, we can calculate the velocity at the exit under a) via application of the energy equation:

\[ C_p T_0 + \frac{1}{2} V_0^2 = C_p T_e + \frac{1}{2} V_e^2 \]

Since \( V_0 = 0 \), we find:

\[ V_e^2 = 2C_p (T_0 - T_e) \]

or

\[ V_e = \sqrt{2C_p (T_0 - T_e)} \quad (1) \]

since only \( T_0 \) is given, we have to find \( C_p \) and \( T_e \).

How to calculate \( T_e \) is already shown.

To calculate \( C_p \), we write

\[ R = C_p - C_v \quad (2) \]

with

\[ \frac{C_p}{C_v} \] transforms in

\[ C_p = \frac{R_f}{f^{-1}} \quad (3) \]

with \( R = 3348 \, J/\text{kg} \cdot \text{K} \) and \( f = 1.26 \) we find

\[ C_p = 1831.85 \, J/\text{kg} \cdot \text{K} \]

with \( T_e = 1715.7 \, \text{K} \) we find

\[ V_e = \sqrt{2 \times 1831.85 \times (3000 - 1715.7)} \]

\[ = 2169.2 \, \text{m/s} \]
Problem 5

a) Compressibility can be neglected when \( M < 0.3 \)

\[
M = \frac{V}{a} \quad V = 245 \text{ km/hr} = 68.06 \text{ m/s}
\]

\[ a = 328.55 \text{ m/s} \]

\[
M = \frac{68.06}{328.55} = 0.207 < 0.3 \quad \text{Incompressible!}
\]

b) Bernoulli along a streamline gives:

\[
P_0 + \frac{1}{2} \rho V_0^2 = P_A + \frac{1}{2} \rho V_A^2
\]

\[
P_0 = 70.121 \text{ N/m}^2
\]

\[
\rho = 0.90926 \text{ kg/m}^3
\]

\[
V_A = 85 \text{ m/s}
\]

\[
P_A = P_0 + \frac{1}{2} \rho (V_0^2 - V_A^2)
\]

\[
= 70.121 + 0.5 \times 0.90926 (68.06^2 - 85^2)
\]

\[
= 68.942.22 \text{ N/m}^2
\]
\[ C_{p_m} = \frac{P_m - P_0}{\frac{1}{2} \rho V^2} = \frac{6894.22 - 70121}{0.5 \times 0.90926 \times 60.66^2} \]
\[ = -0.560 \]

\( a) \quad C_f = 3 \text{ m} \quad C_t = 2 \text{ m} \quad \{ S = 50 \text{ m}^2 \}
\]
\[ b = 20 \text{ m} \quad \rightarrow \quad A = \frac{b^2}{4} = 25 \text{ m}^2 
\]
\[ A = \frac{b^2}{50} \quad \rightarrow \quad A = \frac{400}{50} = 8 \text{ m}^2 
\]

Since this is a wing we write
\[ C_D = C_d + \frac{C_e^2}{\text{RAE}} \]
where \( C_D \) is the profile drag coefficient. The drag force comes from
\[ D = C_D \cdot \frac{1}{2} \rho V^2 S \]

First we have to calculate the wing lift coefficient \( C_L \). With the information we have from the profile,

The profile lift gradient is given by:
\[ a_0 = \frac{(0.97 - 0)}{(6 - (-28))} = \frac{0.97}{0.08} = 0.11 \text{ deg}^{-1} \]
For a wing we know that the lift gradient is given by:

$$ a_0 = \frac{C_D}{1 + \frac{57.3 \times a_0}{\pi \times A_e}} $$

It follows that

$$ a = \frac{0.11}{1 + \frac{57.3 \times 0.11}{\pi \times 0.9 \times 0.9}} = 0.0855 $$

The lift coefficient of the wing follows from:

$$ C_L (4 \text{ deg}) = C_L (-2.5 \text{ deg}) + a(4 - (-2.5)) $$

$$ C_L (4) = 0 + 0.0855 \times 6.8 = 0.5814 $$

Then:

$$ C_{D_{4\text{ deg}}} = 0.0065 + \frac{0.5814^2}{\pi \times 0.9 \times 0.9} $$

$$ = 0.0065 + 0.0149 $$

$$ = 0.02144 $$
The drag force at 4 degrees follows from:

\[ D = 0.02144 \times \frac{1}{2} \times 0.90926 \times 6806^2 \times 50 \]

\[ = 2258.0 \text{ N} \]

e) The lift-drag ratio of the wing \( \frac{C_L}{D} \)
can also be written as \( \frac{C_L}{C_D} \)

\[ C_L \text{ at 4 degrees is 0.5014} \]
\[ C_D \text{ at 4 degrees is 0.02144} \]

\[ \frac{C_L}{C_D} = \frac{0.5014}{0.02144} = 23.5 \]