Introduction to Aerospace Engineering

Lecture slides
Intro to Aerospace Engineering
AE1101ab-3-4 The Standard Atmosphere

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http://natrium42.com/halo/flight2/

30 km hoogte = hoeveel % vd atmosfeer onder je

Link to video at 30 km

Tim Zaman’s (student) project

(h = 30 489 m, 100 020 ft)
Joe Kittinger: jump from 100,000 ft
“Jump from space”?
Why a standard atmosphere?

ISA is reference atmosphere for:

- Meaningful aircraft performance specification
- Pressure altitude definition & EAS/IAS/TAS definition
- Model atmosphere for simulation & analysis
International Standard Atmosphere (ISA)

Layers in the ISA

<table>
<thead>
<tr>
<th>Layer</th>
<th>Level Name</th>
<th>Base Geopotential Height $h$ (in km)</th>
<th>Base Geometric Height $z$ (in km)</th>
<th>Lapse Rate (in °C/km)</th>
<th>Base Temperature $T$ (in °C)</th>
<th>Base Atmospheric Pressure $p$ (in Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Troposphere</td>
<td>0.0</td>
<td>0.0</td>
<td>$-6.5$</td>
<td>$+15.0$</td>
<td>101,325</td>
</tr>
<tr>
<td>1</td>
<td>Tropopause</td>
<td>11.000</td>
<td>11.019</td>
<td>$+0.0$</td>
<td>$-56.5$</td>
<td>22,632</td>
</tr>
<tr>
<td>2</td>
<td>Stratosphere</td>
<td>20.000</td>
<td>20.083</td>
<td>$+1.0$</td>
<td>$-56.5$</td>
<td>5,474.9</td>
</tr>
<tr>
<td>3</td>
<td>Stratosphere</td>
<td>32.000</td>
<td>32.162</td>
<td>$+2.8$</td>
<td>$-44.5$</td>
<td>868.02</td>
</tr>
<tr>
<td>4</td>
<td>Stratopause</td>
<td>47.000</td>
<td>47.350</td>
<td>$+0.0$</td>
<td>$-2.5$</td>
<td>110.91</td>
</tr>
<tr>
<td>5</td>
<td>Mesosphere</td>
<td>51.000</td>
<td>51.413</td>
<td>$-2.8$</td>
<td>$-2.5$</td>
<td>66.939</td>
</tr>
<tr>
<td>6</td>
<td>Mesosphere</td>
<td>71.000</td>
<td>71.802</td>
<td>$-2.0$</td>
<td>$-58.5$</td>
<td>3.9564</td>
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<tr>
<td>7</td>
<td>Mesopause</td>
<td>84.852</td>
<td>86.000</td>
<td>—</td>
<td>$-86.2$</td>
<td>0.3734</td>
</tr>
</tbody>
</table>

\[ T = T_0 + a(h - h_0) \]

\[ a = \text{lapse rate} \]

\[ p = \rho RT \]

\[ dp = -\rho g \, dh \]
Watch out when using lapse rate $a$, it is often given per km, but you should use SI-units, so per m!

International Standard Atmosphere
(= ICAO Std Atm)

Use this instead of Anderson page 112!)

$T = 288.15 \, K \, (15 \, ^{\circ}C)$

$\rho_0 = 101325 \, Pa$

$\rho = 1.225 \, kg/m^3$

$R = 287.05 \, J/kg \, K$

$g_0 = 9.81 \, m/s^2$

(and no water vapour)

$M = 28.97 \, g/mol$
What do we need to define a standard atmosphere?

- Physically correct:
  - Pressure increases due to gravity
  - Gas law

- Two laws, while three variables define state:
  - Pressure
  - Temperature
  - Density

- So by defining one state variable, we define the entire atmosphere by applying the two laws of nature
Hydrostatic equation / geopotential altitude

Equilibrium of forces:

\[ p = p + dp + \rho g dh_g \Rightarrow \]

\[ dp = -\rho g dh_g \quad (1) \]

\[ dp = -\rho g_0 dh \quad (2) \]

This is the one we normally use. Difference is small.

e.g. 63500 ft vs 63307 ft = 0,3%
Absolute altitude & geometric altitude

Geometric altitude: real altitude with sea level = 0

Absolute altitude: distance to centre of earth

\[ h_G = h_a + R_{earth} \]

\[ R_{earth} = 6357 \text{ km} \]
Relation geopotential & geometric altitude

\[ g = g_0 \left( \frac{r}{h_a} \right)^2 = g_0 \left( \frac{r}{r + h_G} \right)^2 \] (3.1)

\[ dp = -\rho g \, dh_G = -\rho g_0 \, dh \quad \rightarrow \quad 1 = \frac{g_0}{g} \frac{dh}{dh_G} \]

\[ dh = \frac{g}{g_0} \, dh_G \] (3.4)

Eq. (3.1) into (3.4):

\[ dh = \frac{r^2}{(r + h_G)^2} \, dh_G \] (3.5)

\[ \int_0^h dh = \int_0^{h_G} \frac{r^2}{(r + h_G)^2} \, dh_G = r^2 \int_0^{h_G} \frac{dh_G}{(r + h_G)^2} \]

\[ h = r^2 \left( \frac{-1}{r + h_G} \right)_0^{h_G} = r^2 \left( \frac{-1}{r + h_G} + \frac{1}{r} \right) = r^2 \left( \frac{-r + r + h_G}{(r + h_G)r} \right) \]

Thus,

\[ h = \frac{r}{r + h_G} \] (3.6)
International Standard Atmosphere (ISA) Layer with T gradient

At sea level:

\[ p_s = 1.01325 \times 10^5 \text{ N/m}^2 \]
\[ \rho_s = 1.225 \text{ kg/m}^3 \]
\[ T_s = 288.15 \text{ K} \]

Temperature in the troposphere (lower part):

\[ \frac{dT}{dh} = -0.0065 \text{ K/m} \]

When \( T = h \) is known as a function of the altitude, the pressure and the density can be derived as a function of altitude:

\[ \frac{p}{p_1} = \left( \frac{T}{T_1} \right)^{-g_0/aR} \]
\[ \frac{\rho}{\rho_1} = \left( \frac{T}{T_1} \right)^{-((g_0/aR)+1)} \]

\( R = \text{gas constant} \)

As an exercise:
try to making an Excel sheet with a table for steps of 100 m
Layer with constant temperature $T$
(11 km - 20 km)

Use values at 11 km as base 1 for this formulae

\[
\frac{p}{p_1} = \frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT}(h-h_1)}
\]

On exam you should be able to derive all ISA formulae!