1. First century of flight: ballooning

1.0 History of ballooning

The twentieth century is often called “the century of flight”. But before the Wright brothers flew their Wright flyer in 1903, mankind had already been flying for 120 years, more than a century, using balloons. For a long time people thought that this was the only way in which humans could fly.

The principle of hot air balloons was already known to the Chinese in the year 300 AD as an invention by Zhuge Liang. The so-called Kong-Ming lanterns were used primarily for military purposes, probably for communication purposes. Today Kong-Ming lanterns are still widely used in Asia on festivals and in many other celebrations. They are also sometimes seen in Europe (and are responsible for numerous UFO reports).

Surprisingly enough, as far as we know, the Chinese never considered to use a larger version to lift a human. The first ones to seriously consider manned flight with a balloon were two French brothers, Joseph-Michel Montgolfier and Jacques-Étienne Montgolfier. In the end of the year 1782 they started experimenting with larger hot air balloons.

The first results were impressive. The lift force that could be generated by increasing the volume of the hot air balloons was larger than they had expected. Consider the effect of scale, of magnification on the ratio of volume (gas, lift) and area (balloon surface, hence the weight).

In 1783 the Montgolfier brothers continued their experiments. The first living creatures to lift off in a man-made device were a sheep, a rooster and a duck. This was not a random collection of a nearby farm. It was part of a scientific experiment to explore whether humans would be able to breathe and survive a flight higher in the atmosphere, away
from the earth surface. Birds that were able to fly could do this, so the duck was a control sample to see whether the craft itself perhaps had an effect. Whether it was a matter of accommodation or a difference between the physiology of birds and mammals had to be tested by including a rooster, a bird that did not fly, and the sheep, as a mammal and model human. All animals survived the flight in good health. (Well, the rooster had a broken wing, because it had been kicked by the sheep)

These experimental flights had by now drawn an immense attention of the crowd and the nobility. Even the king, Louis XVI witnessed the balloon flight with the animals. The next step was to have humans on board. For this the king proposed to use prisoners as test subjects. However, there were some noblemen who volunteered to be the first humans to fly. One of them was Pilâtre de Rozier. a 26 year old physician, who conducted a series of tethered flight experiments before the first real free flight on November 21st, 1783. This first real, because untethered, flight was done together with an army officer, the Marquis d’Arlandes.

Pilâtre de Rozier continued as a flight pioneer. He set a record for speed (70 km/hr), altitude (3000 m) and distance travelled (52 km) in 1784. In 1785 Pilâtre de Rozier died when he crashed during an attempt to cross the English Channel.

Many other pioneering flights and new designs followed. Here you can see an experiment by Guyton de Morveau: a balloon with rudders attached to the craft to control the direction of flight. Why doesn’t this work? (And hence don’t we see this today on balloons)?

Today hot air balloons are still called Montgolfières and the hybrid of a gas and hot air balloon is called a Rozière, after its inventor who used hydrogen for the first time.
1.1 Equation of state instead of gas law

1.1.1 Why we need the equation of state

Below is the ideal gas law. But not in a very useful form for aeronautics because it applies to one amount of gas (n mole with a volume of V). Both n and V are hard to define if you speak about changes of pressure and temperature in the atmosphere.

\[ p \cdot V = n \cdot \mathcal{R} \cdot T \]

*with* \( \mathcal{R} = 8.3145 \ \text{J/molK} \)

Therefore, we will rewrite the ideal gas law to a different form. This new form is called the *equation of state*. The advantage is that it does not have the volume or n in it. By using the air density instead, we can use it to describe any *point* in a flow or in the atmosphere. With the volume that would be much harder.

\[ p \cdot V = n \cdot \mathcal{R} \cdot T \quad \Rightarrow \quad p = \rho \cdot R \cdot T \]

*with* \( R = 287.0 \ \text{J/kgK} \) for air

1.1.2 Derivation of the equation of state from ideal gas law

Since we normally in the atmosphere do not know the volume V nor the number of moles n, we want to get rid of the volume V and n, and use the density instead. This density is defined as:

\[ \rho = \frac{m}{V} \quad \Rightarrow \quad V = \frac{m}{\rho} \]

and:

\[ n = \frac{m}{M} \quad \text{(mass divided by mass per mole)} \]

Now we replace n and V by these expressions:

\[ p \cdot V = n \cdot \mathcal{R} \cdot T \quad \Leftrightarrow \quad (\text{substitute } V \text{ and } n) \]

\[ p \cdot \frac{m}{\rho} = \frac{m}{M} \cdot \mathcal{R} \cdot T \quad \Leftrightarrow \quad (\text{divide left and right side by } m) \]

\[ \frac{p}{\rho} = \frac{\mathcal{R}}{M} \cdot T \quad \Leftrightarrow \quad (\text{multiply by } \rho) \]

\[ p = \rho \cdot \frac{\mathcal{R}}{M} \cdot T \quad \Leftrightarrow \quad (\text{redefine } R = R_{\text{air}} = \frac{\mathcal{R}}{M_{\text{air}}} = 287 \ \text{J/kgK}, \text{ with } M_{\text{air}} = 0.02897 \ \text{kg/mol}) \]
We will use this equation of state for many purposes. But we have to remember that the $R$ now has become dependent on the molar mass of the gas. So the value 287 is only valid for air. We have to keep in mind the definition of $R$:

$$R = \frac{\mathcal{R}}{M}$$

$$R_{air} = \frac{\mathcal{R}}{M_{air}} = 287 \tfrac{J}{kg K}, \text{ with } M_{air} = 0.02897 \tfrac{kg}{mol}.$$
1.3 Aerostatics: How does a balloon fly?

1.3.1 Principle
When studying aerospace, you expect a lot of aerodynamics (and you will not be disappointed). But just like flight started with balloons, we start our discussion with aerostatics. Aerostatics is the name for flying with balloons, called in contrast with aerodynamics for the effect of moving air, which we will cover later.

Consider these questions:

- What generates more lift: a helium filled balloon or a hot air balloon?
- How many helium-filled party balloons are required to lift a human?
- Is the rigid airship the green aircraft of the future?

Archimedes (287 BC – 212 BC) was a brilliant physicist, mathematician and inventor living in the Greek colony Syracuse on Sicily. He is regarded by many as the best scientist of the antiquity. Unfortunately he was killed by a hot-headed roman soldier. He was the first to understand the physical principle of floating. Take a bath with water.

Consider a certain smaller volume of this water. What are the forces on this volume? First of all there is the gravity, which can be written as a function of the volume $V$, the gravity constant $g$ (9,81 m/s$^2$) and the density $\rho$.

$$F_G = m \cdot g = \rho \cdot V \cdot g \quad \text{(down)}$$

Because the water is in equilibrium, the net resulting force of the surrounding water on this volume must be equal to this force but then in the reverse direction: up. This is called the buoyant force or Archimedes force:

$$F_A = \rho \cdot V \cdot g \quad \text{(up)}$$

The balloons exploit the same principle as floating on water does (then called hydrostatics). But it uses a gas, lighter than air: hot air or another chemical composition.

Using this knowledge we can calculate the lift force a balloon can generate.
The lift can be calculated using the difference between the weight of the balloon and the weight of the air that would have been in this volume, had there not been a balloon.

Without the balloon, when there is air in this volume, there is an equilibrium. This means that the sum of all forces around this volume has to be equal to the weight of this air. These atmospheric forces do not change when there is balloon in this volume, and the weight of the balloon is different from (less than) the weight of the air.

Therefore when there is a lighter craft like a balloon in this volume, we can calculate the lift by simply calculating the difference between the weight of the craft and the air it replaces:

\[ L_G = W_{air} - W_{gas} \]

\[ L_G = \rho_{atm} V g - \rho_{gas} V g \]

\[ L_G = \rho_{atm} V g \left(1 - \frac{\rho_{gas}}{\rho_{atm}}\right) \]

This true for both balloon types: gas and hot air. But it is also not a very practical form. Rarely will we know the density of the gas or hot air.

For a hot air balloon we would like to know the lift depending on the increase of temperature and the volume. And for the gas balloon, we would like to know the volume as a function of the selected gas type.
1.3.2 Lift of hot-air balloons

Let us start with a hot air balloon. The interior is $\Delta T$ degrees hotter than the outside air in the atmosphere. Let us use the above equation with densities as a starting point:

\[
L_G = \rho_{\text{atm}} \, V \, g \left( 1 - \frac{\rho_{\text{hot air}}}{\rho_{\text{atm}}} \right)
\]

and

\[
p = \rho \cdot R \cdot T \quad \Leftrightarrow \quad \rho = \frac{p}{R \cdot T}
\]

This yields:

\[
L_G = \rho_{\text{atm}} \, V \, g \left( 1 - \frac{p}{R \cdot T_{\text{hot}}} \right)
\]

\[
L_G = \rho_{\text{atm}} \, V \, g \left( 1 - \frac{T_{\text{hot}}}{T_{\text{atm}}} \right)
\]

\[
L_G = \rho_{\text{atm}} \, V \, g \left( 1 - \frac{T_{\text{atm}}}{T_{\text{hot}}} \right)
\]

Often it is more convenient to use $\Delta T$, the increase in temperature, compared to the outside temperature,

\[
L_G = \rho_{\text{atm}} \, V \, g \left( 1 - \frac{T_{\text{atm}}}{T_{\text{atm}} + \Delta T} \right)
\]

\[
L_G = \rho_{\text{atm}} \, V \, g \left( \frac{T_{\text{atm}} + \Delta T}{T_{\text{atm}} + \Delta T} - \frac{T_{\text{atm}}}{T_{\text{atm}} + \Delta T} \right)
\]
This is an equation that we can use. We know the density and temperature of the atmosphere. When we then need a certain lift we can calculate the required temperature for a given volume of the balloon. Then we can for example see whether that is realistic, (temperature should be less than 120 °C) or whether we need a bigger balloon.

1.3.3 Lift of gas balloons

For gas balloons we use the same equation as a start.

\[ L_G = \rho_{am} V g \left( 1 - \frac{\rho_{gas}}{\rho_{am}} \right) \]

In hot air balloon the pressure and molar mass was the same inside and outside the balloon. With gas balloons the pressure and temperature are equal. One could argue that the pressure inside might be a little higher as a result of the tension of the balloons material. This is in the equations neglected below. In larger balloons, like weather balloons, the volume of the balloon is much larger than the volume of the gas when lifting off, to allow expansion when reaching heights with a lower pressure. In that case assuming an equal pressure is acceptable.

Using the assumption of equal pressure and temperatures we can write for the ratio of the densities:

\[ p = \rho \cdot \frac{9R}{M} \cdot T \iff \rho = \frac{p \cdot M}{9R \cdot T} \iff \frac{\rho_1}{\rho_2} = \frac{M_1}{M_2} \]

If we substitute this in the equation above, we get:

\[ L_G = \rho_{am} V g \left( 1 - \frac{M_{gas}}{M_{air}} \right) \]
To get a feeling for these numbers, here are the molar masses of some gasses:

- Helium: \( M_{He} = 4.00 \text{ g/mol} \)
- Hydrogen: \( M_{H_2} = 2.02 \text{ g/mol} \)
- Air (averaged): \( M_{air} = 28.97 \text{ g/mol} \)

So for Helium the latter part of the equation becomes:

\[
L_G = \rho_{atm} V g \left( 1 - \frac{M_{He}}{M_{air}} \right) = \rho_{atm} V g \left( 1 - \frac{4.00}{28.97} \right) \approx \rho_{atm} V g \cdot \frac{6}{7}
\]

If we compare this to hot-air balloons:

\[
L_G = \rho_{atm} V g \frac{\Delta T}{T_{atm} + \Delta T}
\]

This implies that \( \Delta T \) should be six times as high as the \( T_{atm} \) in Kelvin. So for an outside air temperature of 15 °C (288 K), this would mean an air temperature approaching the 1800 °C! This is hotter than a blowtorch! This clearly shows that in terms of generation of lift, Helium is better. Which reasons can you think of why the majority of balloons uses hot air despite this lower efficiency?

Using the formulae we can calculate the lifting capacity per cubic metre of gas at average conditions. For this we assume an air density of 1.225 kg/m\(^3\) and an air temperature of 15 °C (288 K):

1 m\(^3\) Helium: \[
L_G = \rho_{atm} V g \left( 1 - \frac{M_{He}}{M_{air}} \right) = 1.225 \cdot 1.00 \cdot 9.81 \left( 1 - \frac{4.00}{28.97} \right) \approx 10 N
\]

1 m\(^3\) 120 °C air: \[
L_G = \rho_{atm} V g \frac{\Delta T}{T_{atm} + \Delta T} = 1.225 \cdot 1.00 \cdot 9.81 \cdot \frac{105}{288+105} \approx 3.2 N
\]
1.4 Balloon problems

Problem 1:
Typical volume for a 3–4 person hot-air balloon is said to be 2500 m$^3$. What is the total weight of the balloon, basket and payload for such a balloon, assuming the mentioned maximum temperature of 120 ºC. (assume $\rho_{atm} = 1.225$ kg/m$^3$, $T = 15$ ºC, $g=9.81$ m/s$^2$)

Problem 2:
Parameters balloon of first human flight in a Montgolfière are said to be: volume 1700 m$^3$, lifting capacity 780 kg (=balloon, basket and payload, some claim an even higher weight of 830 kg, but let’s now use the smaller figure of 780 kg). Use the standard pressure at sea level: 1013.25 mbar

a) What was the temperature of the balloon? Assume an outside air temperature of 10.0 ºC.

b) Other sources say the balloon was 23 m high and 14 m wide. Calculate the temperature inside the balloon for this situation as well. (Hint: Assume a spherical shape that is elongated in the vertical direction by a factor 23/14).

c) Which source would you believe based on the outcome? What would you estimate the volume of the real Montgolfière to have been?

Problem 3:
How many helium filled party balloons would be required to take-off for a person of 80 kg? A party balloon has a volume of about 14 liters. (use $\rho_{atm} = 1.225$ kg/m$^3$)

Problem 4:
When you get higher in the atmosphere, the air density becomes lower. The balloon will expand because the pressure is also lower. What will happen when the balloon reaches its maximum altitude? What do you need to know to calculate that maximum altitude?

Problem 5:
Consider a gas balloon just after its release at sea level. The balloon does not change shape. This means on every part of the surface of the balloon the forces are in equilibrium, so apparently the inside and outside force are equal. Still, the overall effect is that the balloon wants to accelerate to go up. How is this possible?

*Problem 6:*
What are the advantages of a balloon compared to a winged aircraft in terms of lift? And air drag? So for what types of flight is the balloon more efficient than the winged aircraft?