Problem Set 4.

Ben Polak Econ 159a/MGT522a

Three questions due October 10, 2007

(1) Comparative Statics of Mixed Strategy Equilibria. Consider the following two-player game.

	l	r
U	$12, 2$	$3, 9$
D	$5, 8$	$4, 2$

(a) Find all the Nash equilibria, pure and mixed. Explain how you know you have found all the equilibria.

(b) Suppose that the payoff of the column player $u_2(D, l)$ is reduced from 8 to 6, but all other payoffs remain the same. Again, find all the pure- and mixed-strategy Nash equilibria.

(c) Compare the mixed-strategy equilibria in parts (a) and (b). Did this worsening in one of player 2's payoffs change player 2's equilibrium mixed strategy? Did it change player 1's? Give some intuition.

2. On Her Majesty's Secret Service. The famous British spy 001 has to choose one of four routes a, b, c, or d (listed in order of speed in good conditions) to ski down a mountain. Fast routes are more succeptible to avalanche. At the same time, the notorious rival spy 002 has to choose whether to use ("y") or not to use ("x") his valuable explosive to try to cause an avalanche. The payoffs to this game are as follows.

		002	
		x	y
001	a	$12, 0$	$0, 6$
	b	$11, 1$	$1, 5$
	c	$10, 2$	$4, 2$
	d	$9, 3$	$6, 0$

(a) Let $p_1(x)$ be the probability assigned by 001's belief to 002's playing x. Explain what 001 should do if $p_1(x) > \frac{2}{3}$; if $p_1(x) < \frac{2}{3}$; and if $p_1(x) = \frac{2}{3}$?

(b) Suppose you are Mr. Queue, the Yale-educated technical advisor to British military intelligence. Are there any routes you would advise 001 certainly **not** to take? Explain your answer.

(c) The gripped viewer of this epic drama is trying to figure out what will happen. Find a Nash equilibrium in which one player plays a pure strategy s and the other player plays a mixed strategy μ. Find a different mixed-strategy equilibrium in which that pure strategy s is assigned zero weight? Are there any other equilibria?

3. Sound advice. (Adapted from Osborne.) Suppose that player 1's stereo system is not working properly: it only plays mono. He does not know whether it needs an easy repair (say, a cleaning) or a major overhaul (say, a new laser). The probability that it needs a new laser is ρ. At his local repair store, he finds that a new laser costs L, while a cleaning costs C ($L > C$). He knows that the expert at the store, player 2, gets the same profit, π, if she charges him for a new laser and indeed fixes the laser, or if she charges him for a cleaning and indeed just cleans it. But she can make more profit, $\Pi > \pi$, if she charges him for a new laser but in fact (secretly) just cleans it. If it only needed a cleaning anyway, then she will get away with this, but she knows she will get sent to jail if she only cleans it when it needed a new laser. The expert is very good at her job, so she knows which is needed.

(a) Explain why player 1 should always believe player 2 when she says it just needs a cleaning, but why he might be skeptical if she says it needs a new laser.

Player 1 can reject the expert's advice and get a second opinion from a consultant who never lies. Assume if he does this, however, he must accept the second expert's advice and accept new repair costs, $L' > L$ or $C' > C$. Here then is the game between player 1 (row) and player 2 (column).

	Honesty	Dishonesty
Always accept advice	$-\rho L - (1-\rho)C, \quad \pi$	$-L, \quad \rho\pi + (1-\rho)\Pi$
Reject if told 'laser'	$-\rho L' - (1-\rho)C, \quad (1-\rho)\pi$	$-\rho L' - (1-\rho)C', \quad 0$

(b) Explain why each entry is as it is, in this payoff matrix.

(c) Assume that $L > \rho L' + (1-\rho)C'$. Explain why there is no pure-strategy Nash equilibrium. Give an intuition for this condition.

(d) Find the (unique) mixed-strategy Nash equilibrium; that is, find the equilibrium randomizations in terms of the parameters.

(e) As we increase the cost of a laser repair at the first expert L (holding all the other parameters fixed), what happens to the equilibrium probability that the expert chooses the 'honest' strategy? What happens to the equilibrium probability that player 1 chooses the strategy 'reject if told laser'? Give some intuition.

(f) As we increase the profit from lying Π (holding all the other parameters fixed), what happens to the equilibrium probability that the expert chooses the 'honest' strategy? What happens to the equilibrium probability that player 1 chooses the strategy 'reject if told laser'? Give some intuition.

(g) It has been said that, "in America, when people go to the doctor, they never think they have a cold: they think they have 'mono"'. Assuming this is true, why might we expect doctors in America often to act dishonestly? [Hint: think about how the parameter ρ affects the equilibrium in the above model].