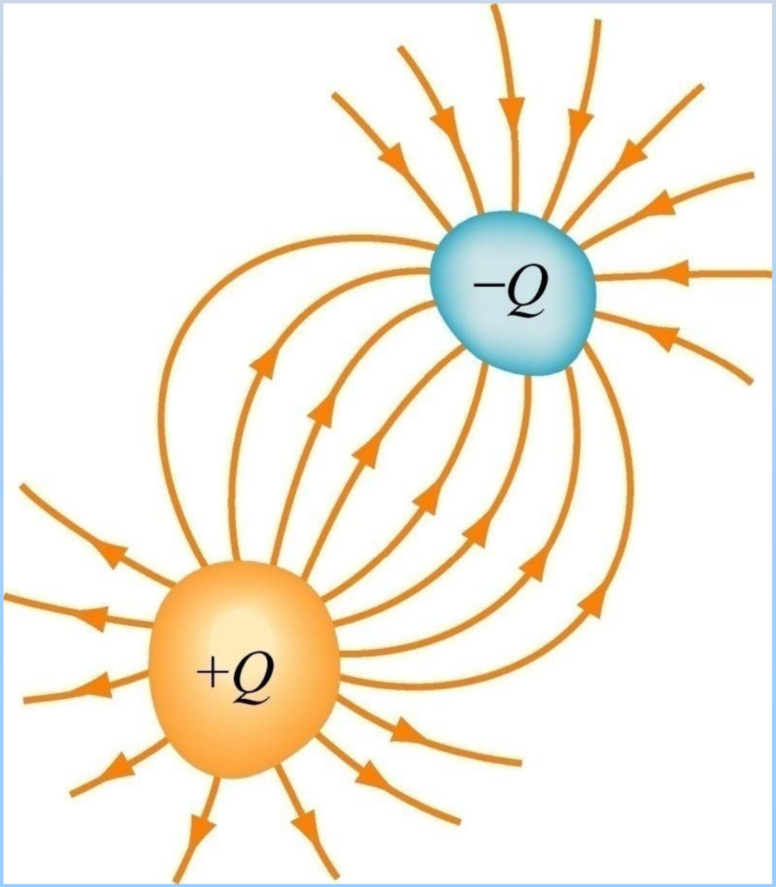
Module 10: Capacitance

Capacitors and Capacitance

Our first of 3 standard electronics devices
(Capacitors, Resistors & Inductors)

Capacitors: Store Electric Charge

Capacitor: Two isolated conductors
Equal and opposite charges ±Q
Potential difference ΔV between them.

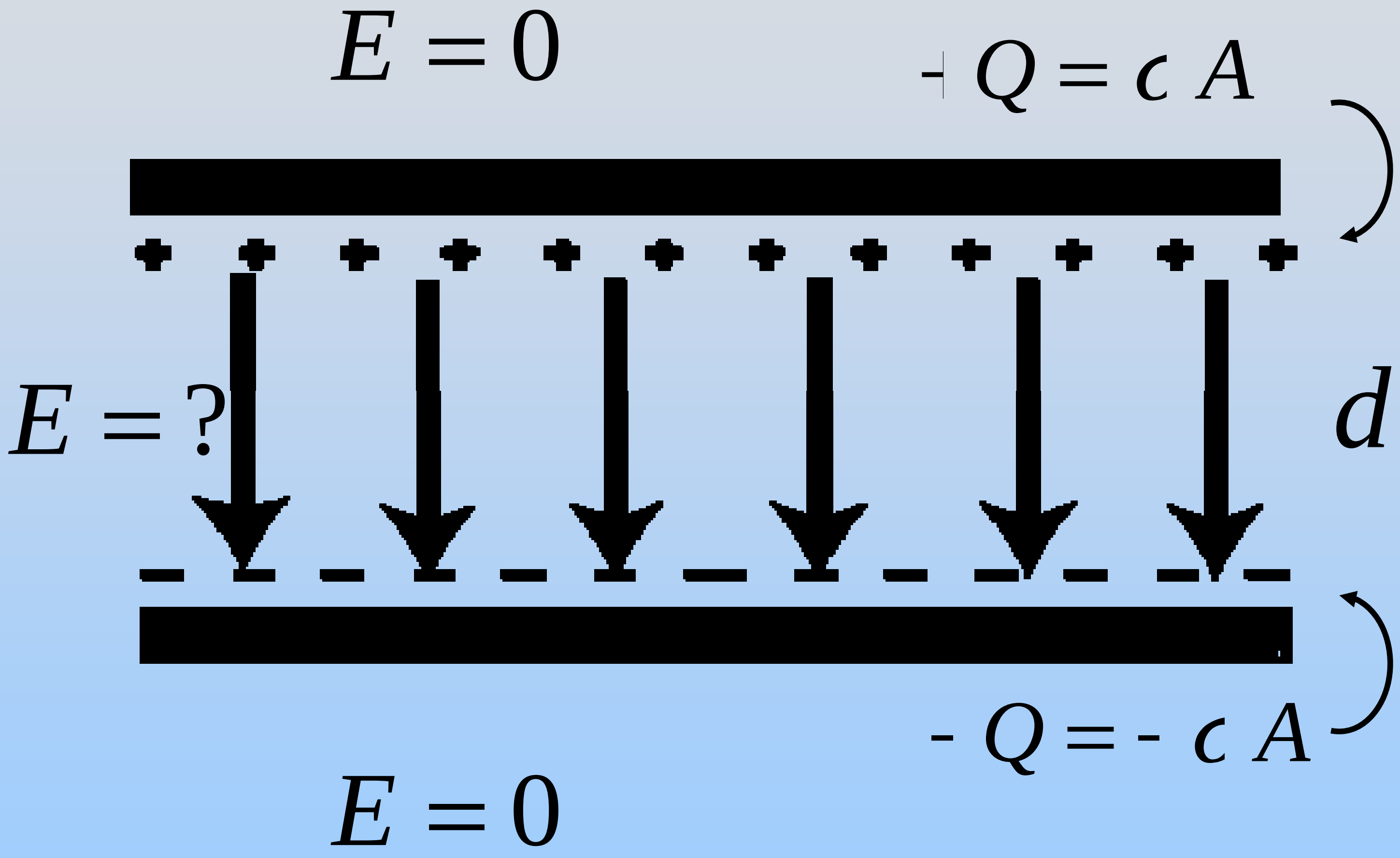


$$\boxed{C = \frac{Q}{|\Delta V|}}$$

Units: Coulombs/Volt or Farads

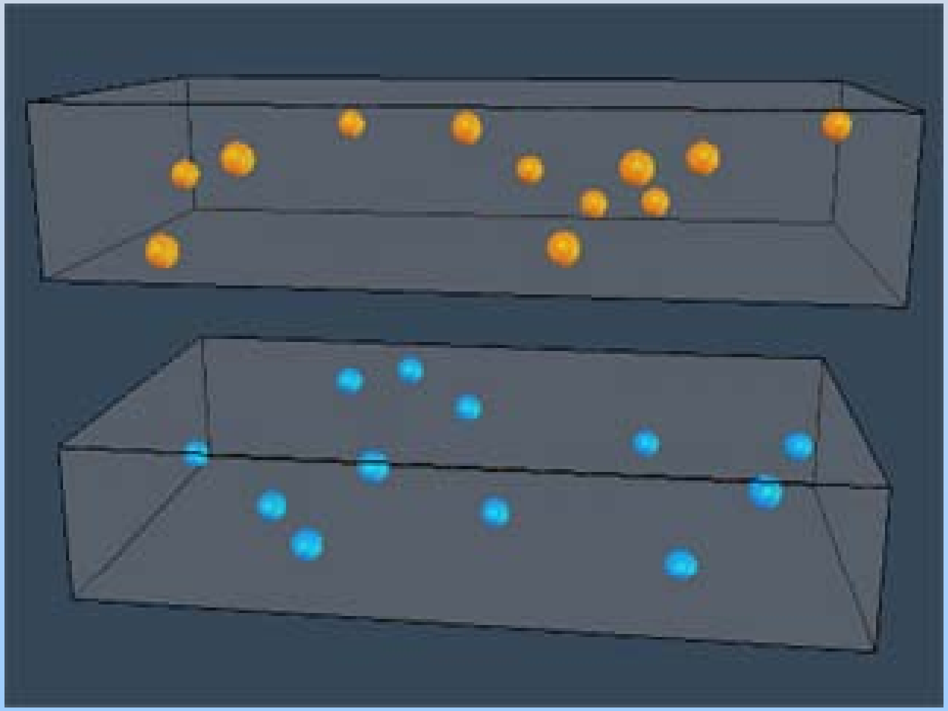
C is Always Positive

Parallel Plate Capacitor



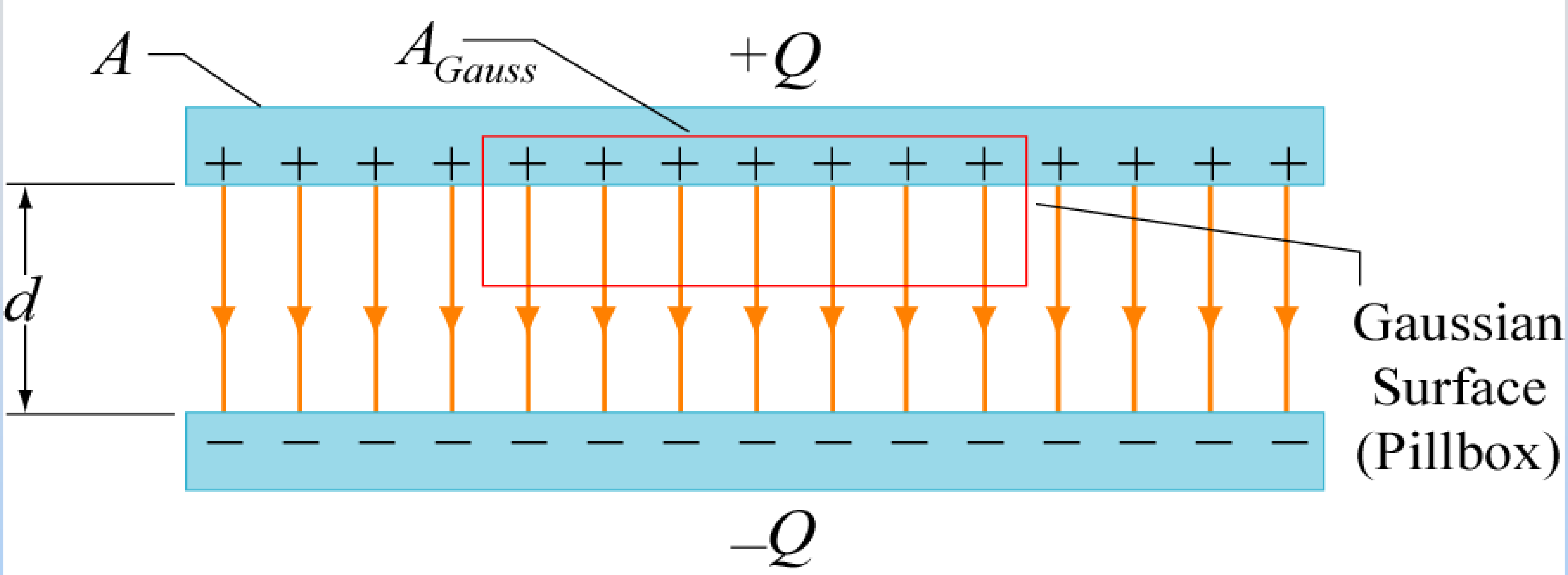
Parallel Plate Capacitor

Oppositely charged plates:
Charges move to inner surfaces to get close



[Link to Capacitor Applet]

Calculating E (Gauss's Law)



$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

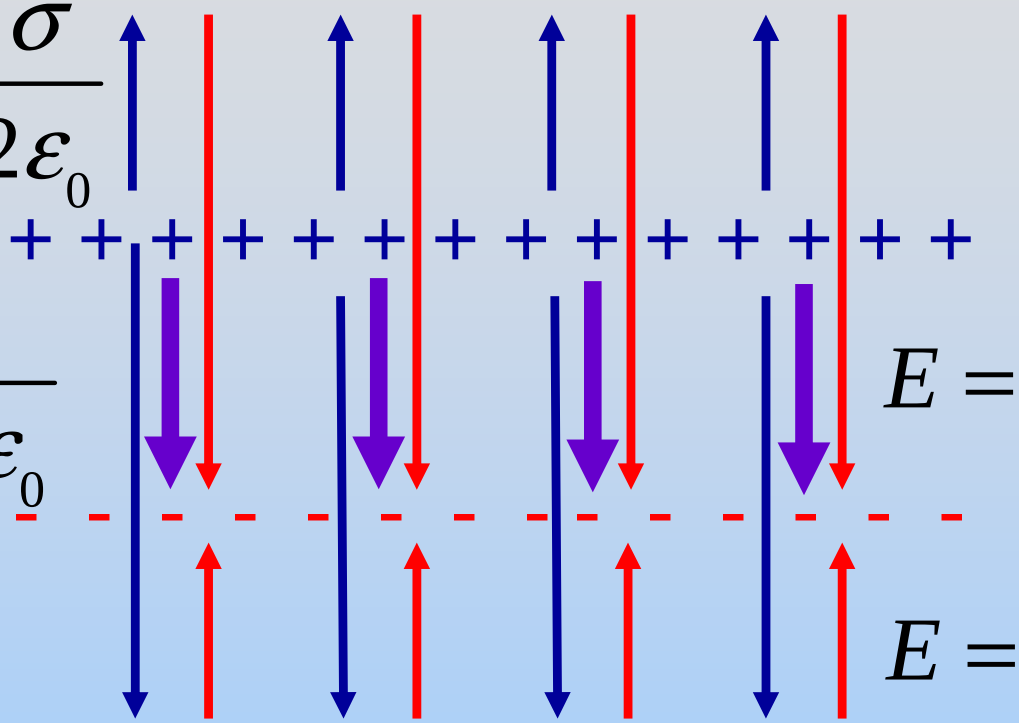
$$E\left(A_{Gauss}\right) = \frac{\sigma A_{Gauss}}{\varepsilon_0}$$

$$\boxed{E = \frac{\varsigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}}$$

Note: We only "consider" a single sheet!

Doesn't the other sheet matter?

Alternate Calculation Method



$$E = - \frac{\sigma}{2\varepsilon_0}$$

Top Sheet:

$$E = \frac{c}{2\varepsilon_0}$$

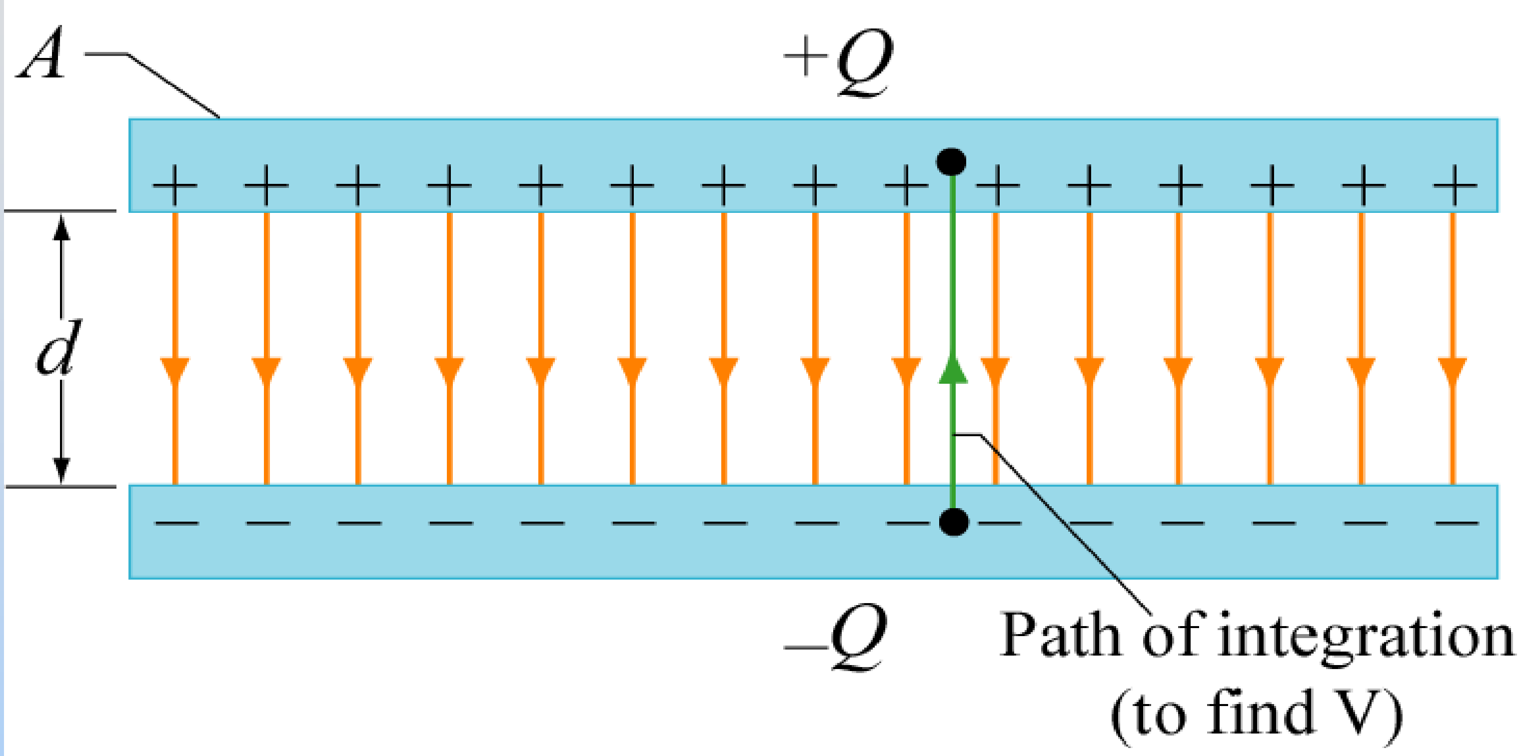
$$E = \frac{c}{2\varepsilon_0}$$

Bottom Sheet:

$$E = - \frac{\sigma}{2\varepsilon_0}$$

$$\boxed{E = \frac{c}{2\varepsilon_0} + \frac{c}{2\varepsilon_0} = \frac{c}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}}$$

Parallel Plate Capacitor



$$\Delta V = -\int_{bottom}^{top} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = Ed = \frac{Q}{A\varepsilon_0} d$$

$$\boxed{C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d}}$$

C depends only on geometric factors A and d

Concept Question Questions: Changing C Dimensions

Concept Question: Changing Dimensions

A parallel-plate capacitor has plates with equal and opposite charges ±Q, separated by a distance *d*, and **is not** connected to a battery. The plates are pulled apart to a distance *D* > *d*. What happens?

1. V increases, Q increases
2. V decreases, Q increases
3. V is the same, Q increases
4. V increases,Q is the same
5. V decreases, Q is thesame
6. V is the same, Q is the same
7. V increases, Q decreases
8. V decreases, Q decreases
9. V is the same,Q decreases

Concept Question: Changing Dimensions

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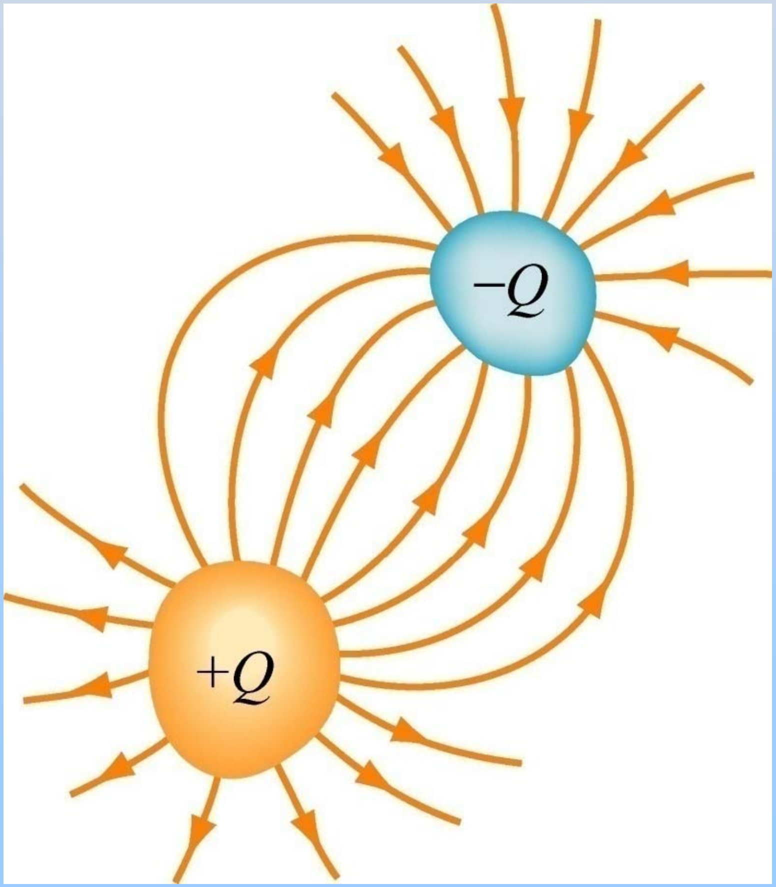
Demonstration:
Changing C Dimensions

Capacitors: Review

Capacitor: Two isolated conductors
Equal and opposite charges ±Q
Potential difference ΔV between them.

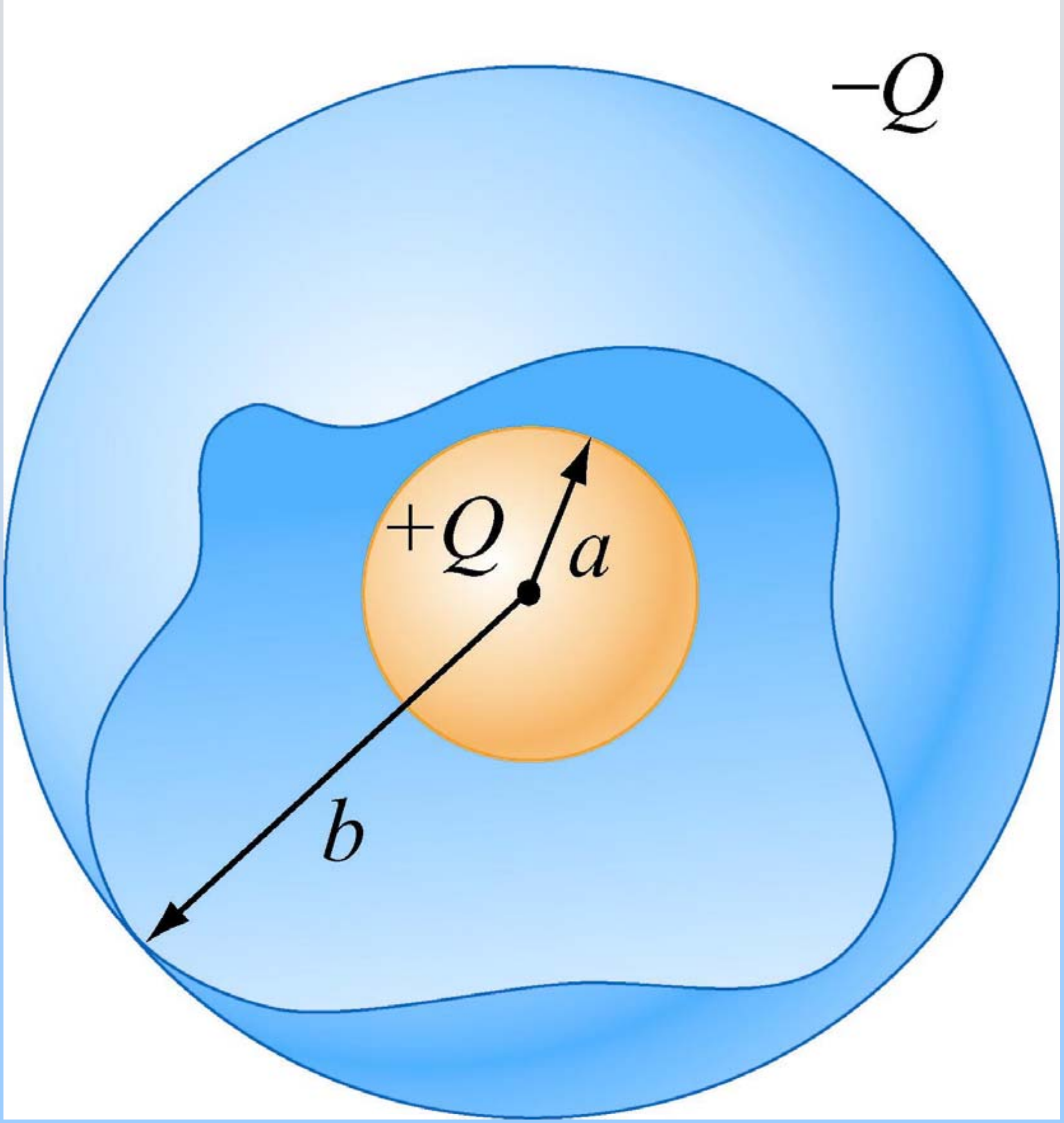


$$\boxed{C = \frac{Q}{\left|\Delta V\right|}}$$

Units: Coulombs/Volt or Farads

C is Always Positive

Group Problem: Spherical Shells



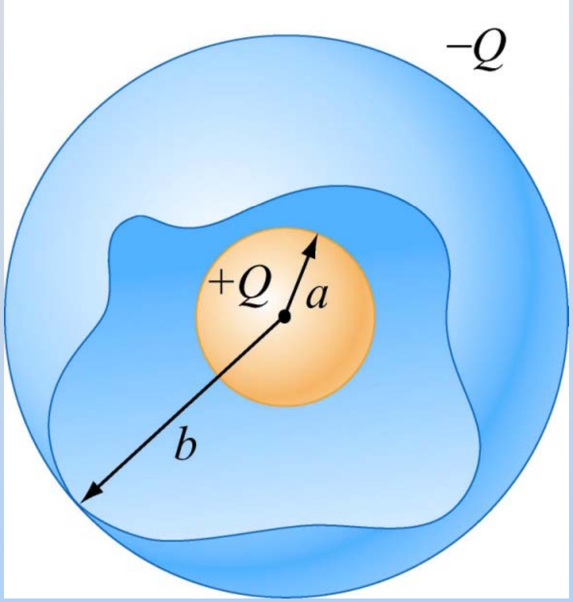
These two spherical shells have equal but opposite charge.

Find E everywhere

Find V everywhere (assume $V(\infty) = 0$)

Spherical Capacitor

Two concentric spherical shells of radii *a* and *b*



What is E?

Gauss's Law → E ≠ 0 only for *a* < *r* < *b*, where it looks like a point charge:

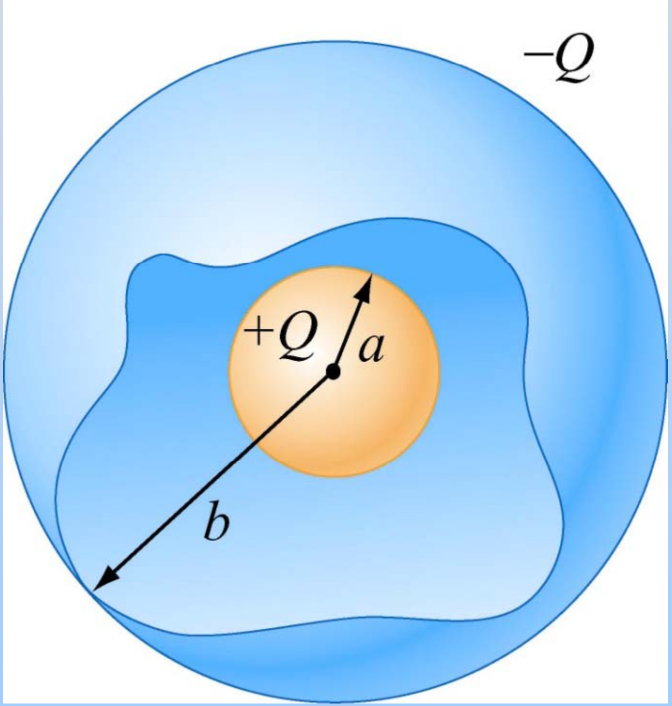
$$\vec{\mathbf{E}} - \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

Spherical Capacitor

$$\Delta V = -\int_{inside}^{outside} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -\int_a^b \frac{Q\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} \cdot dr\,\hat{\mathbf{r}} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

Is this positive or negative? Why?

$$\boxed{C = \frac{Q}{\left|\Delta V\right|} = \frac{4\pi\varepsilon_0}{\left(a^{-1} - b^{-1}\right)}}$$



For an isolated spherical conductor of radius *a:*

$$C = 4\pi\varepsilon_0 a$$

Capacitance of Earth

For an isolated spherical conductor of radius *a:*

$$C = 4\pi\varepsilon_0 a$$

$$\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{F/m} \qquad a = 6.4\times10^{6}\,\mathrm{m}$$

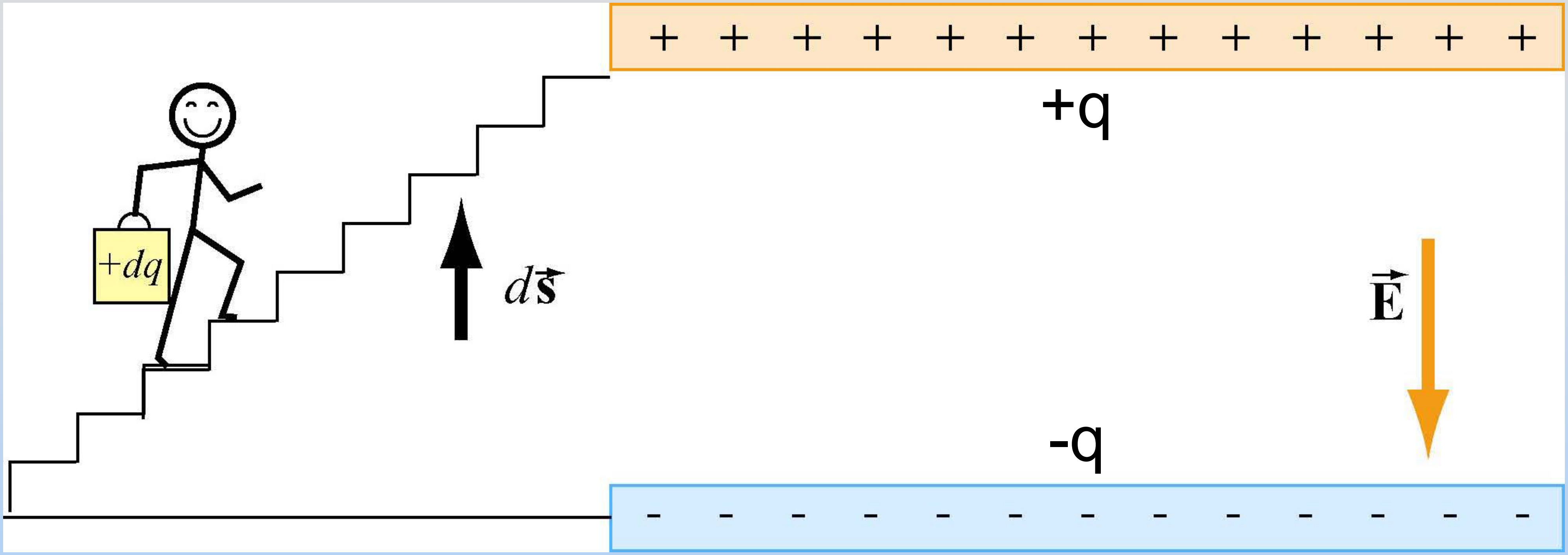
$$C = 7\times10^{-4}\,\mathrm{F} = 0.7\mathrm{mF}$$

A Farad is REALLY BIG! We usually use pF (10^{-12}) or nF (10^{-9})

Energy Stored in Capacitor

Start charging capacitor

Energy To Charge Capacitor



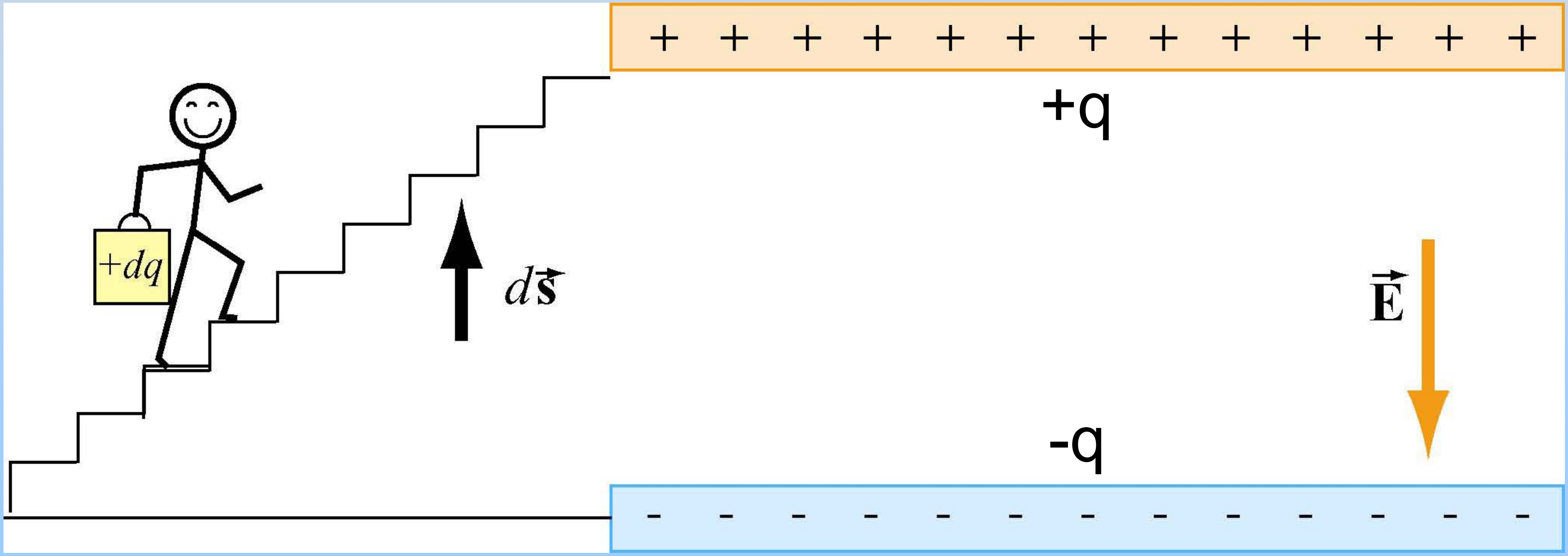
1. Capacitor starts uncharged.
2. Carry +*dq* from bottom to top.
   Now top has charge q = +dq, bottom -dq
3. Repeat
4. Finish when top has charge q = +Q, bottom - Q

Work Done Charging Capacitor

At some point top plate has +q, bottom has –q

Potential difference is $\Delta V = q / C$

Work done lifting another dq is $dW = dq\, \Delta V$



Work Done Charging Capacitor

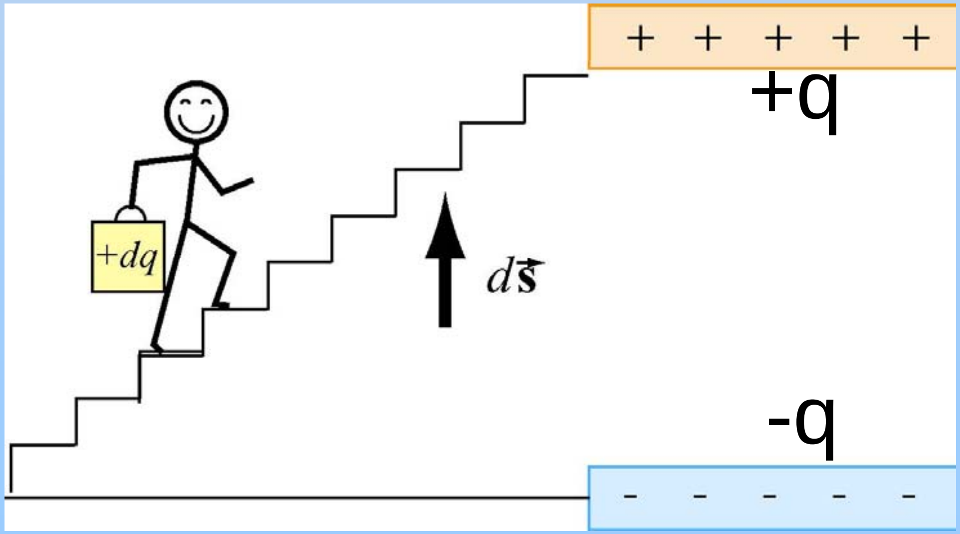
So work done to move *dq* is:

$$dW = dq\,\Delta V = dq\frac{q}{C} = \frac{1}{C}q\,dq$$

Total energy to charge to *q = Q*:

$$W = \int dW = \frac{1}{C}\int_0^Q q\,dq$$

$$\boxed{= \frac{1}{C}\frac{Q^2}{2}}$$



Energy Stored in Capacitor

Since $C = \dfrac{Q}{|\Delta V|}$

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} Q|\Delta V| = \frac{1}{2} C|\Delta V|^2}$$

Where is the energy stored???

Energy Stored in Capacitor

Energy stored in the E field!

Parallel-plate capacitor: $C = \dfrac{\varepsilon_o A}{d}$ and $V = Ed$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_o A}{d}\left(Ed\right)^2 = \frac{\varepsilon_o E^2}{2}\times(Ad) = u_E \times (volume)$$

$$\boxed{u_E = E \text{ field energy density} = \frac{\varepsilon_o E^2}{2}}$$

Concept Question Question: Changing C Dimensions Energy Stored

Concept Question: Changing Dimensions

A parallel-plate capacitor, disconnected from a battery, has plates with equal and opposite charges, separated by a distance d.

Suppose the plates are pulled apart until separated by a distance $D > d$.

How does the final electrostatic energy stored in the capacitor compare to the initial energy?

1. The final stored energy is smaller
2. The final stored energy is larger
3. Stored energy does not change.

Demonstration:
Big Capacitor
Exploding a Wire

8.02SC Physics II: Electricity and Magnetism
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