Module 19:
Sources of Magnetic Fields:
Biot-Savart Law

Module 19: Outline

Magnetic Fields, Creating Fields: Biot-Savart Law

Sources of Magnetic Fields

What creates fields?

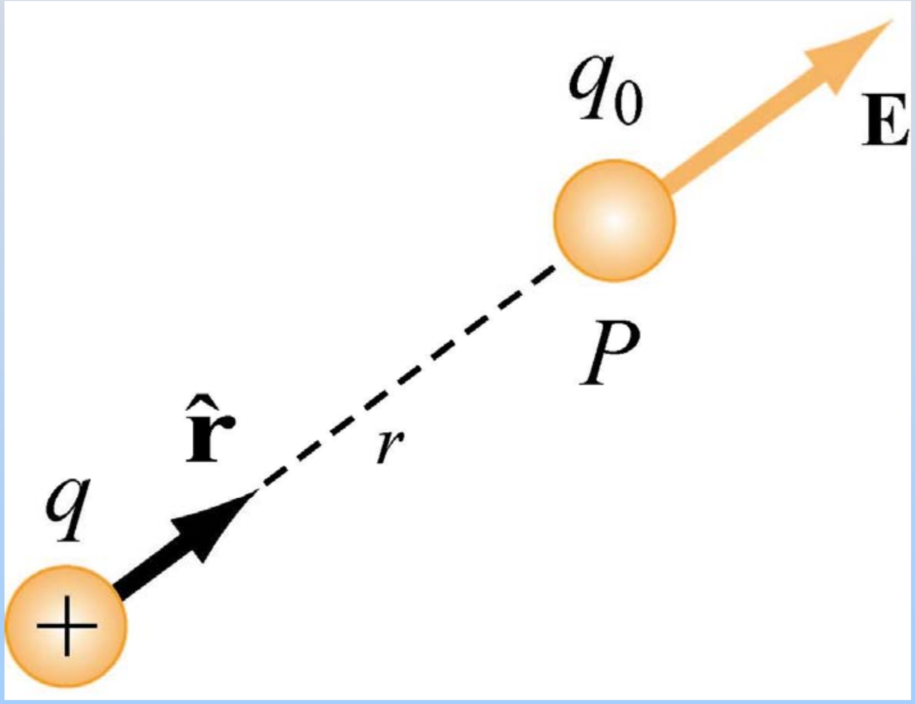
Magnets – more about this later

The Earth How’s that work?

Moving charges!

Electric Field Of Point Charge

An electric charge produces an electric field:

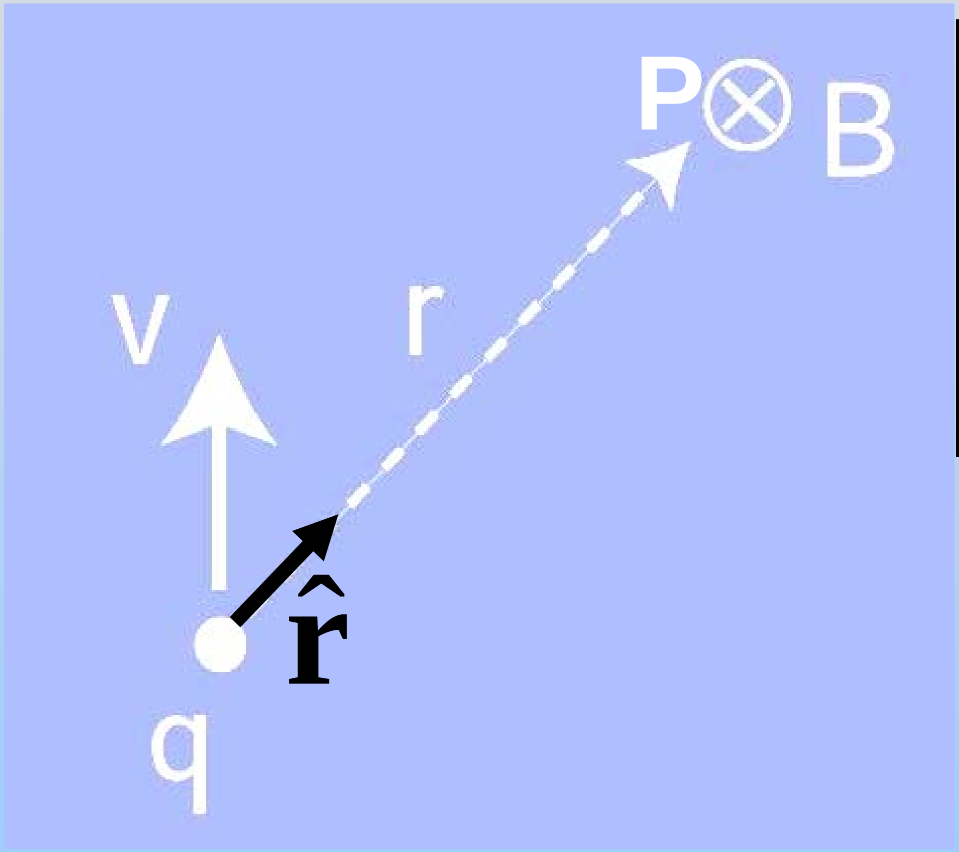


$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{\mathbf{r}}$$

$\hat{\mathbf{r}}$: unit vector directed from q to *P*

Magnetic Field Of Moving Charge

Moving charge with velocity v produces magnetic field:

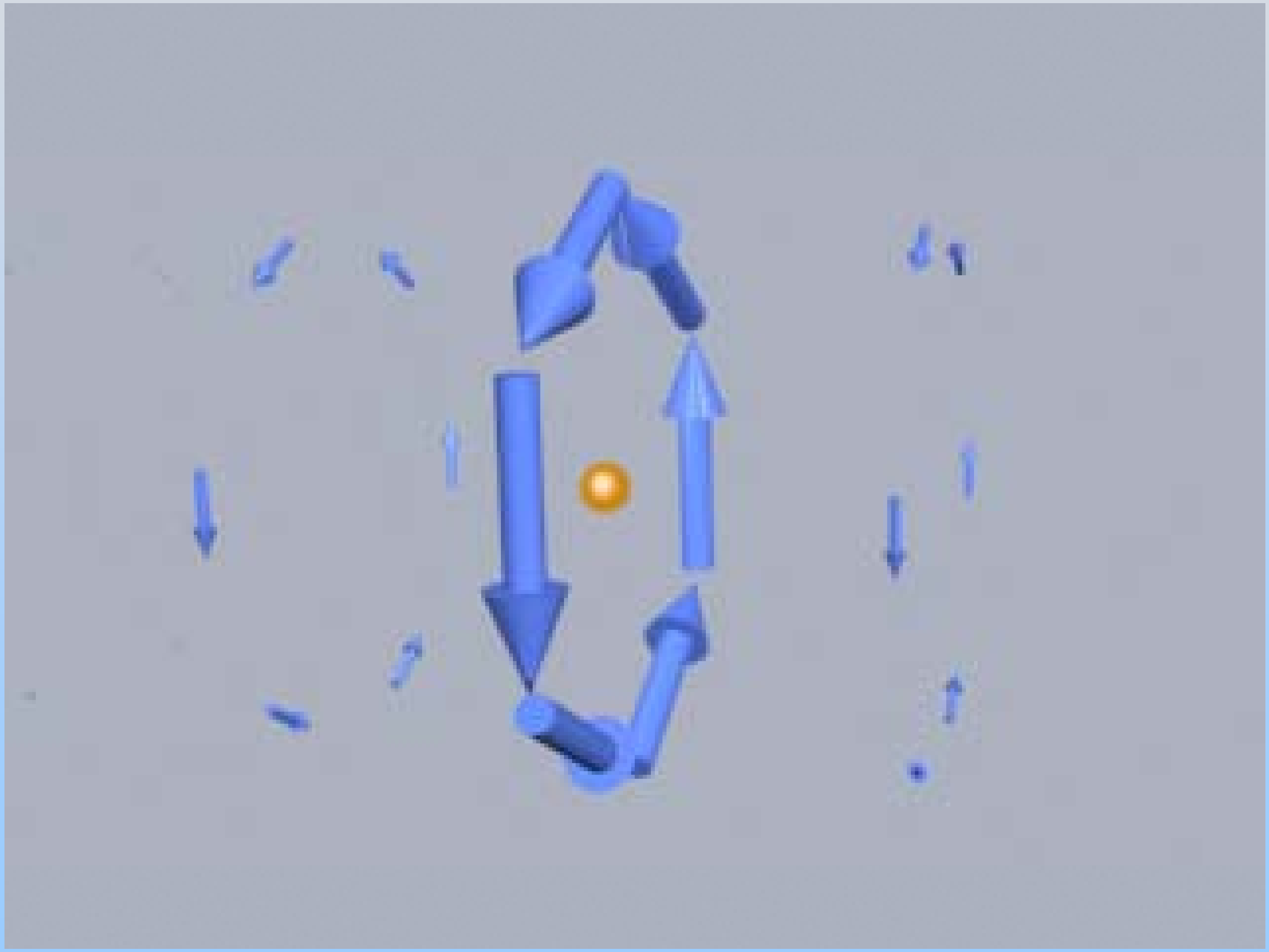


$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

$\hat{\mathbf{r}}$: unit vector directed from q to P

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ permeability of free space

Animation:
Field Generated by a Moving Charge



Link to animation

Moving ^ Continuous charge distributions: Currents & Biot-Savart

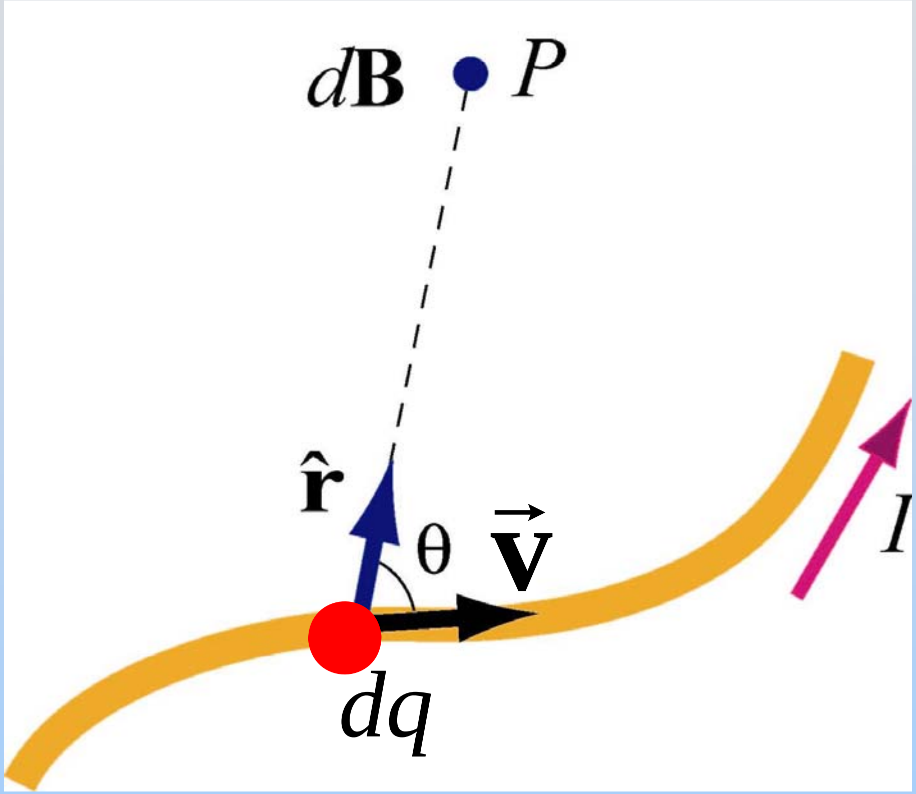
From Charges to Currents?

$$d\vec{\mathbf{B}} \propto dq\,\vec{\mathbf{v}}$$

$$= [\text{coulomb}]\frac{[\text{meter}]}{[\text{sec}]}$$

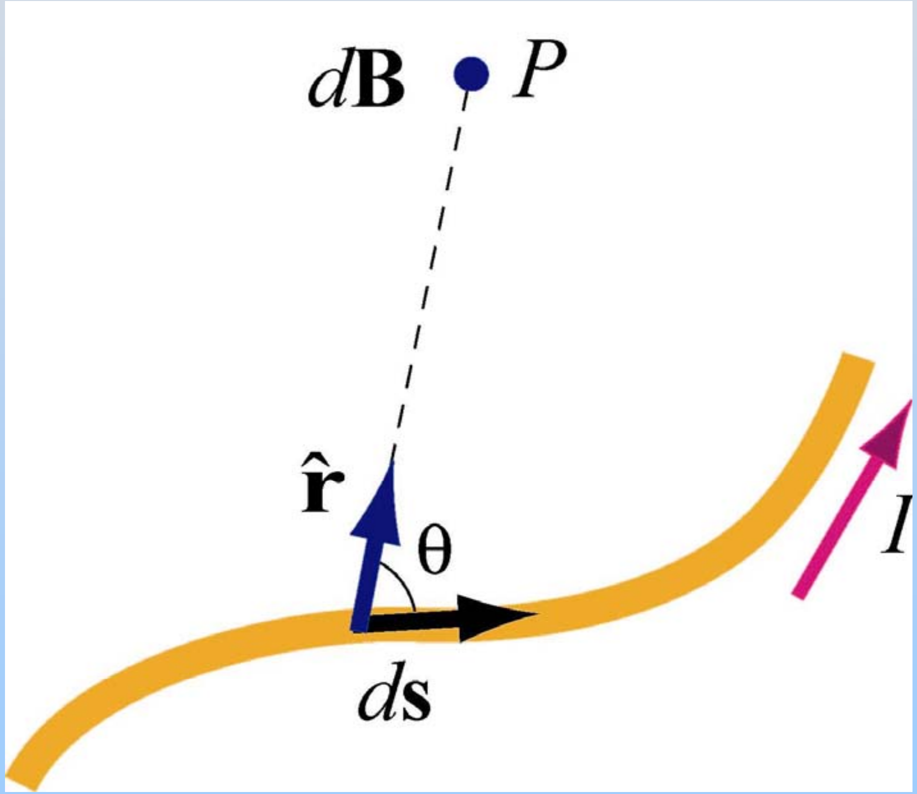
$$= \frac{[\text{coulomb}]}{[\text{sec}]}[\text{meter}]$$

$$\boxed{d\vec{\mathbf{B}} \propto I\,d\vec{\mathbf{s}}}$$



The Biot-Savart Law

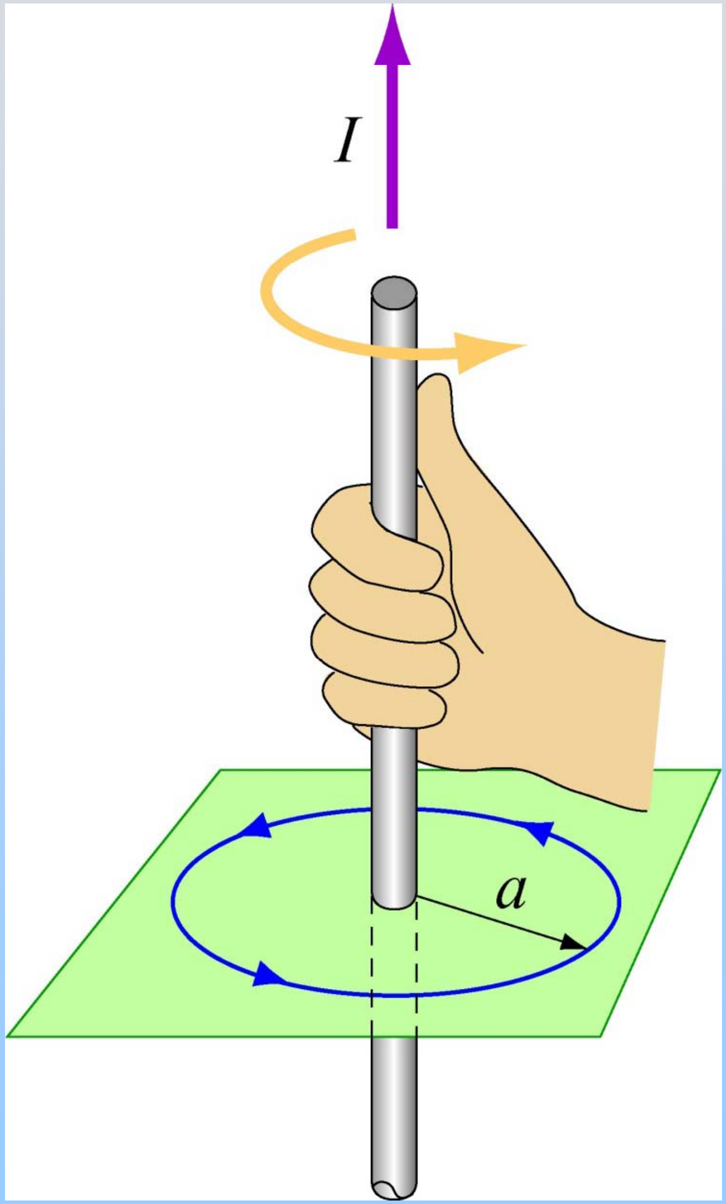
Current element of length ds carrying current I produces a magnetic field:

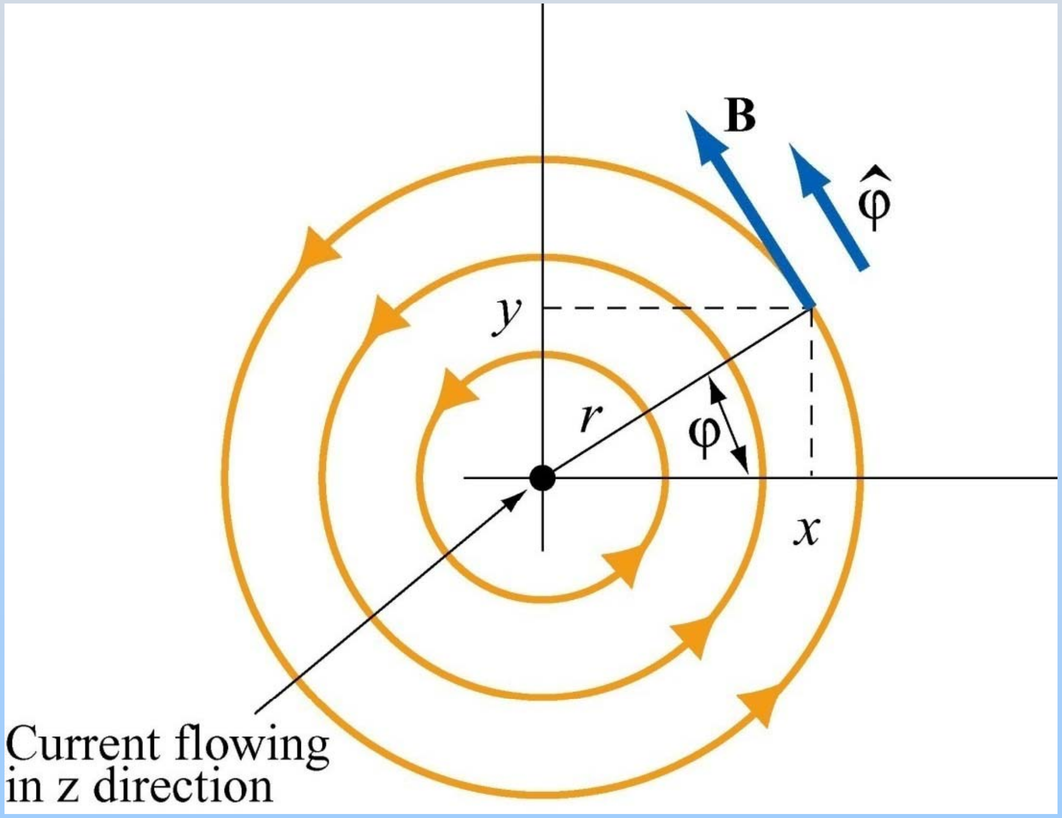


$$\mathbf{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

(Link to Shockwave)

The Right-Hand Rule #2





$$\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\theta}}$$

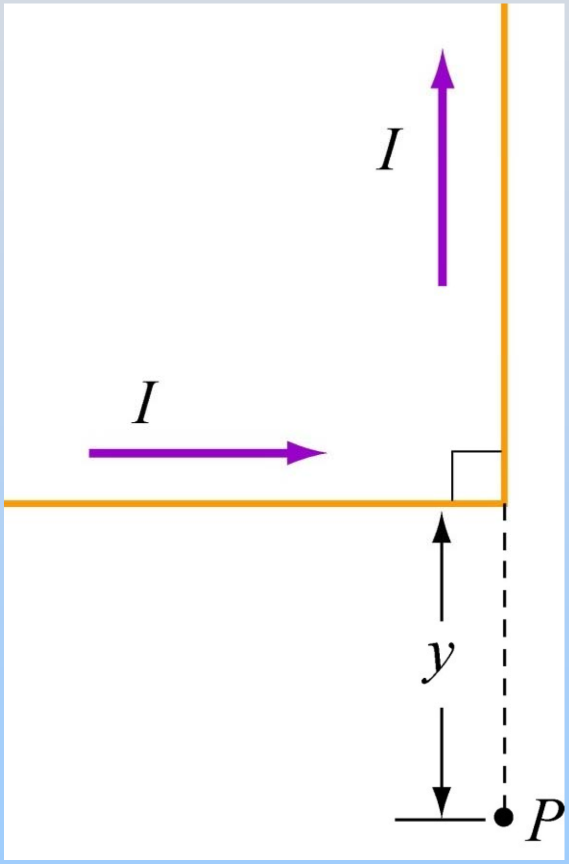
Concept Questions:
B fields Generated by Currents

Concept Question: Biot-Savart

0

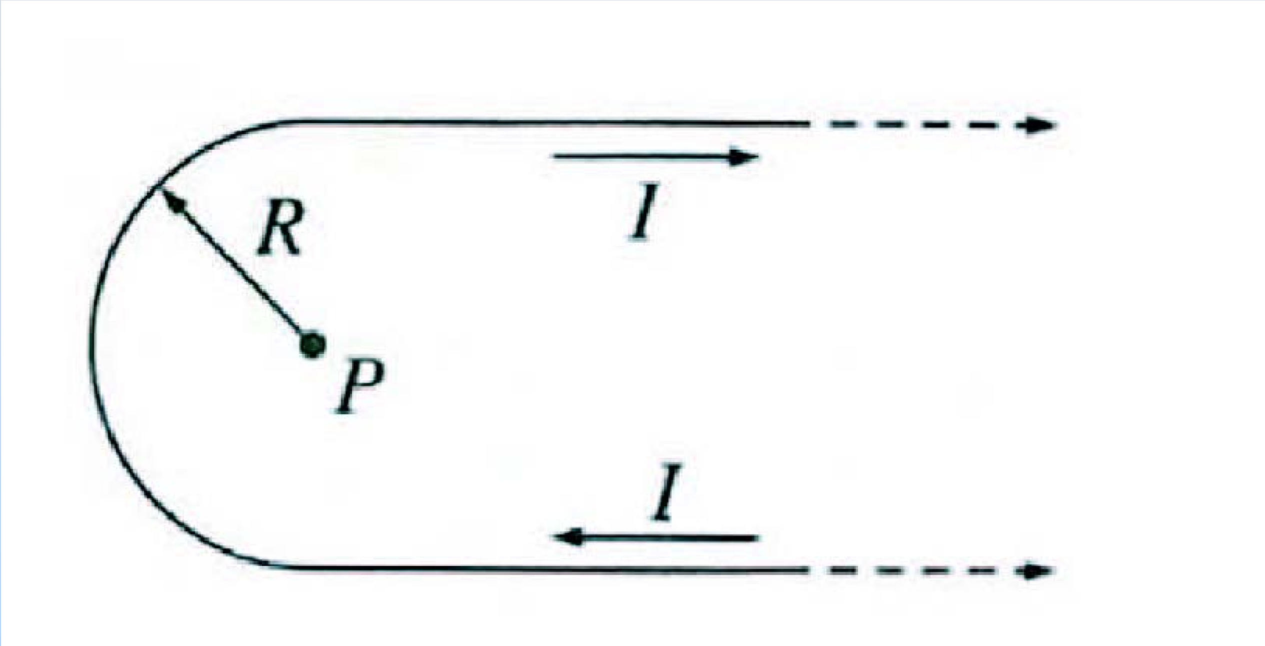
The magnetic field at P points towards the

1. +x direction
2. +y direction
3. +z direction
4. -x direction
5. -y direction
6. -z direction
7. Field is zero (so no direction)



Concept Question: Bent Wire

The magnetic field at P is equal to the field of:

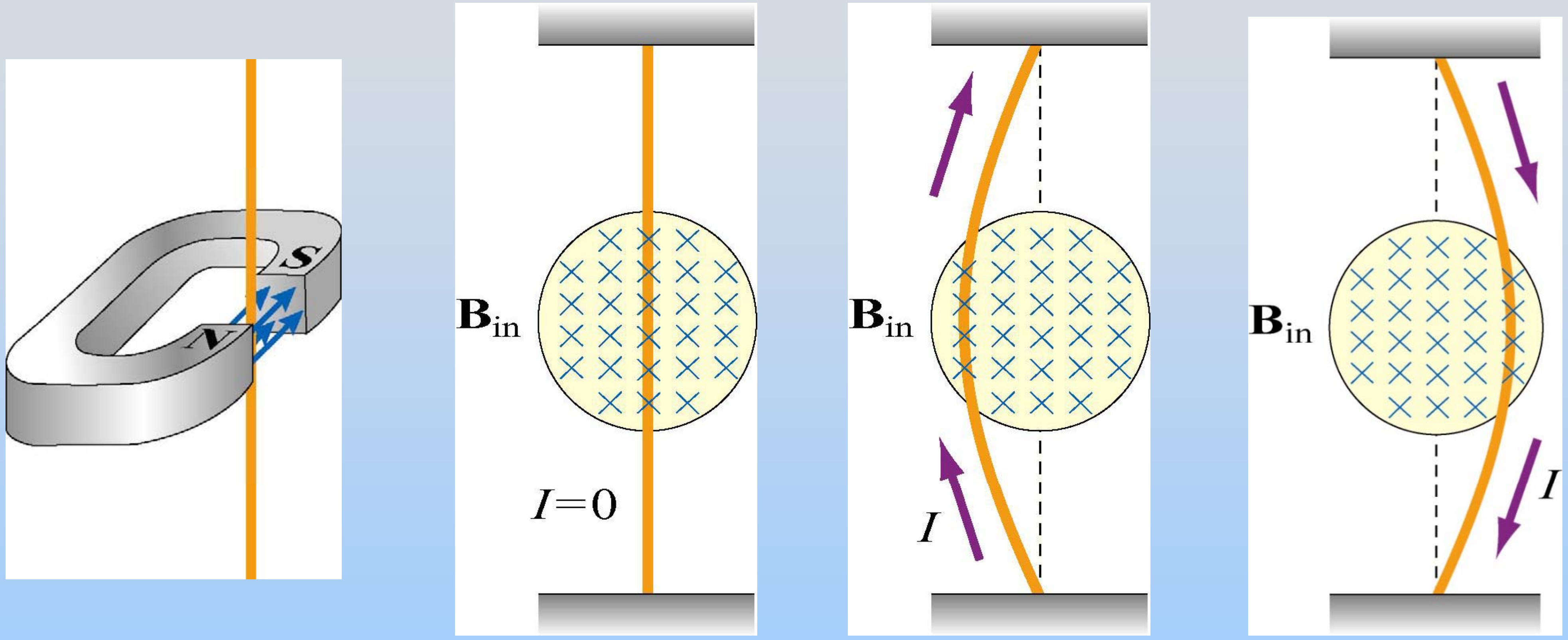


1. a semicircle
2. a semicircle plus the field of a long straight wire
3. a semicircle minus the field of a long straight wire
4. none of the above

Demonstration:
Field Generated by Wire

Demonstration: Jumping Wire

Magnetic Force on Current-Carrying Wire



Current is moving charges, and we know that moving charges **feel** a force in a magnetic field

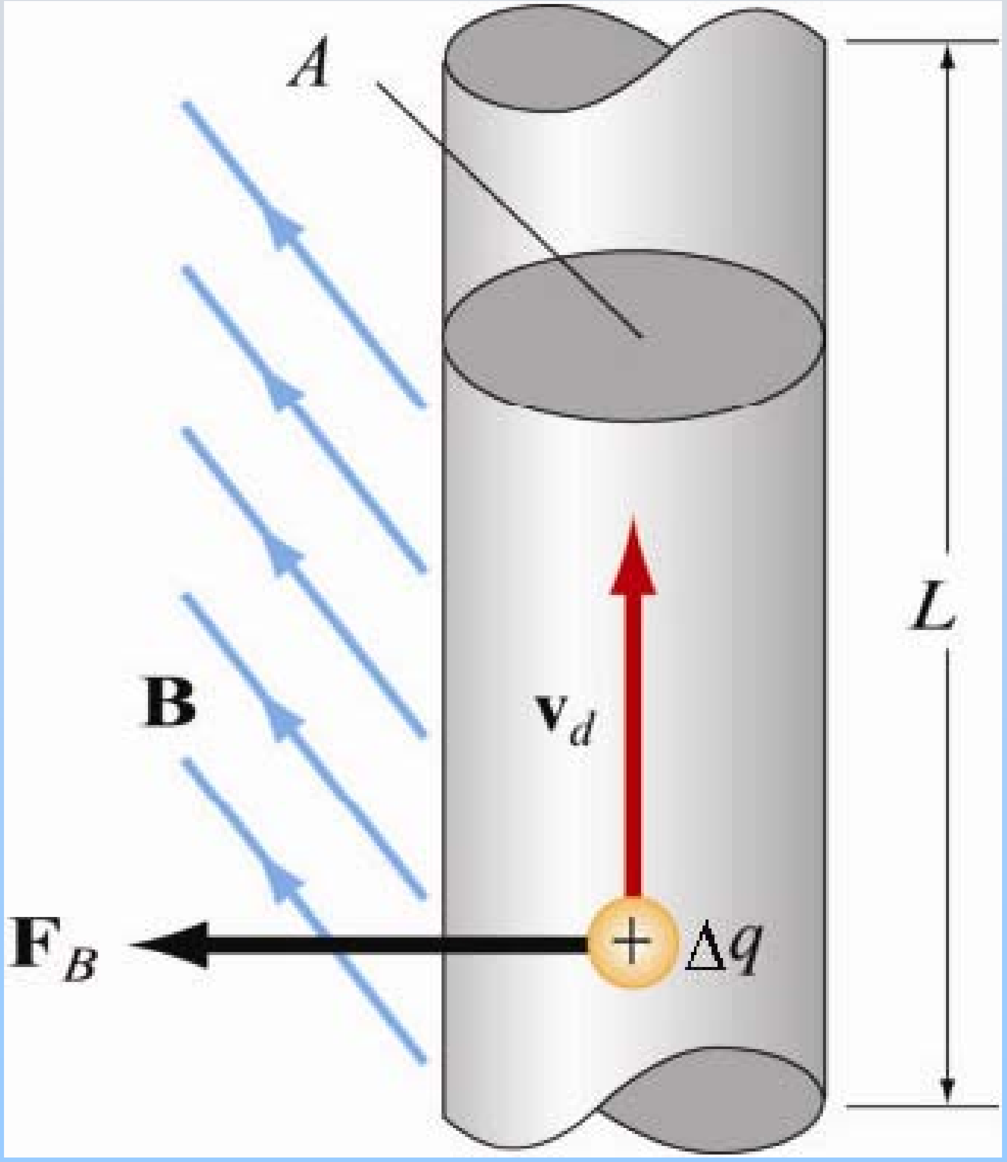
Magnetic Force on Current-Carrying Wire

$$\vec{\mathbf{F}}_B = \Delta q \vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$\Delta q \vec{\mathbf{v}} = \Delta q \frac{\Delta \vec{\mathbf{s}}}{\Delta t}$$

$$= \frac{\Delta q}{\Delta t} \Delta \vec{\mathbf{s}} = I \Delta \vec{\mathbf{s}}$$

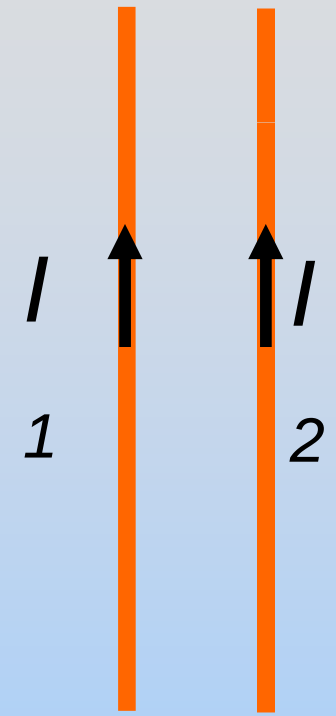
$$\boxed{\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)}$$



Concept Question Question: Parallel Current Carrying Wires

Concept Question: Parallel Wires

Consider two parallel current carrying wires. With the currents running in the same direction, the wires are

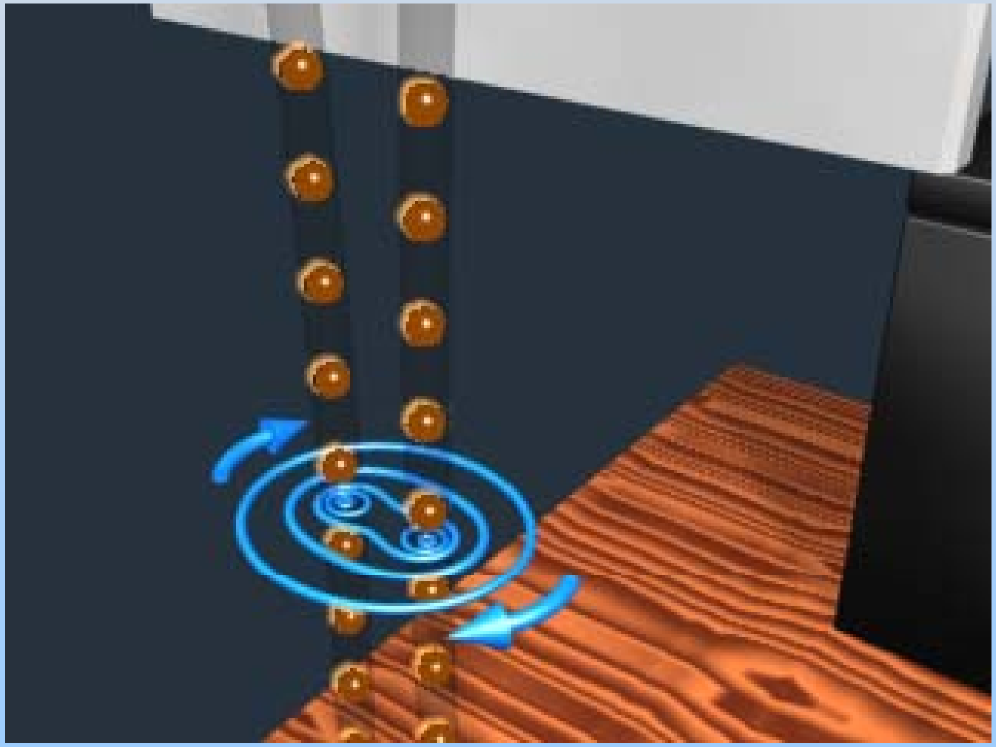


1. attracted (likes attract?)
2. repelled (likes repel?)
3. pushed another direction
4. not pushed – no net force
5. I don't know

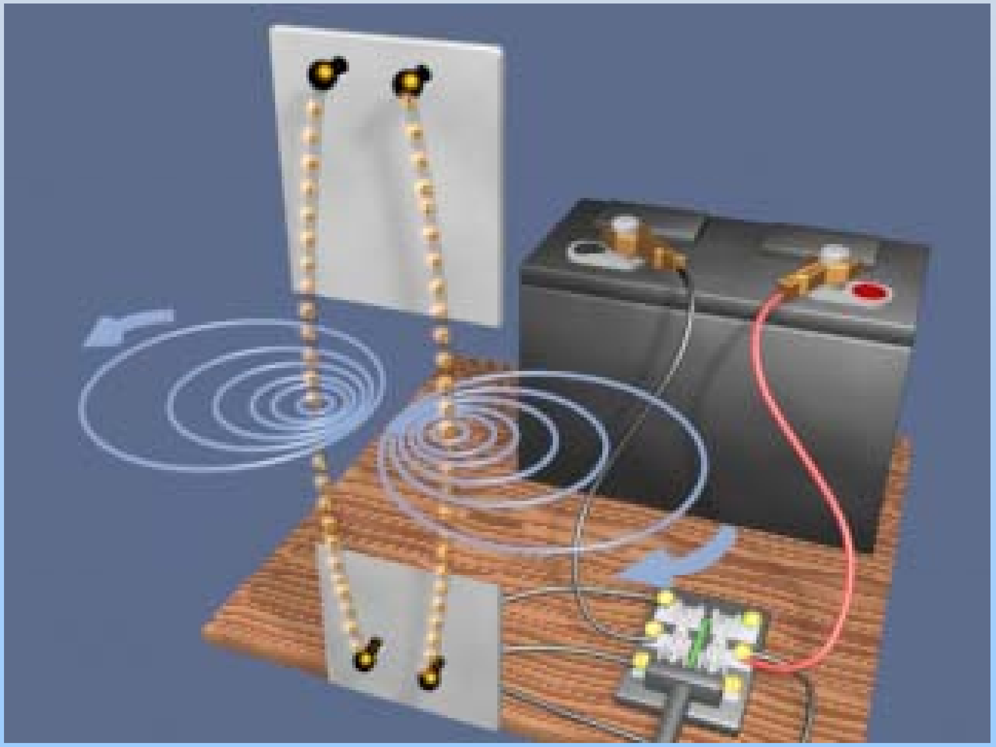
Demonstration:
Parallel & Anti-Parallel Currents

Can we understand why?

Whether they attract or repel can be seen in the shape of the created B field

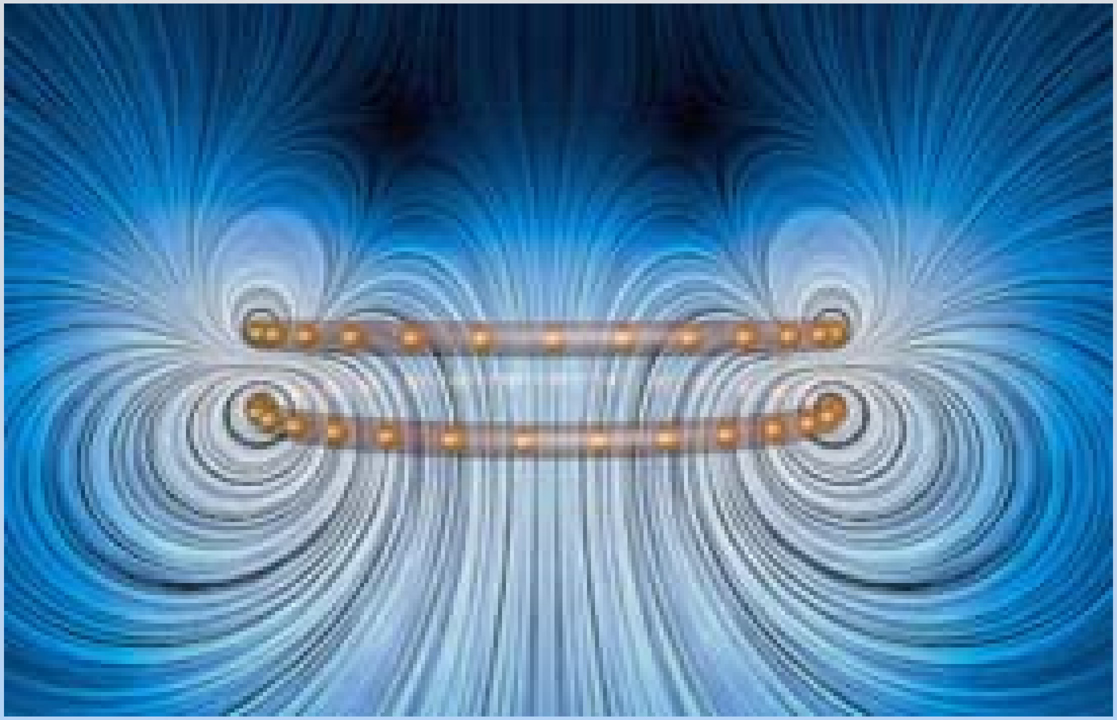


(Link to Animation)



(Link to Animation)

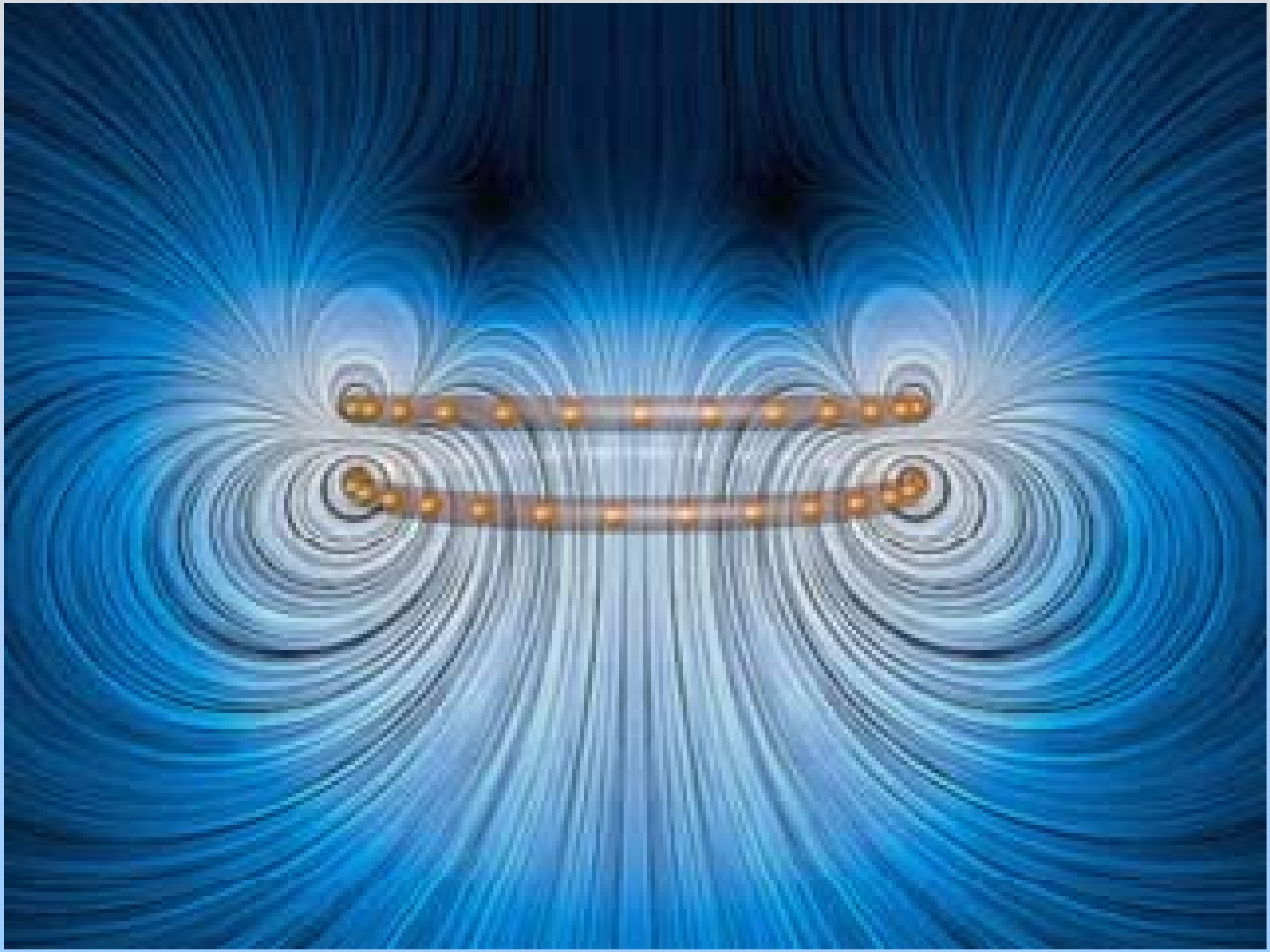
Concept Question: Current Carrying Coils



The above coils have

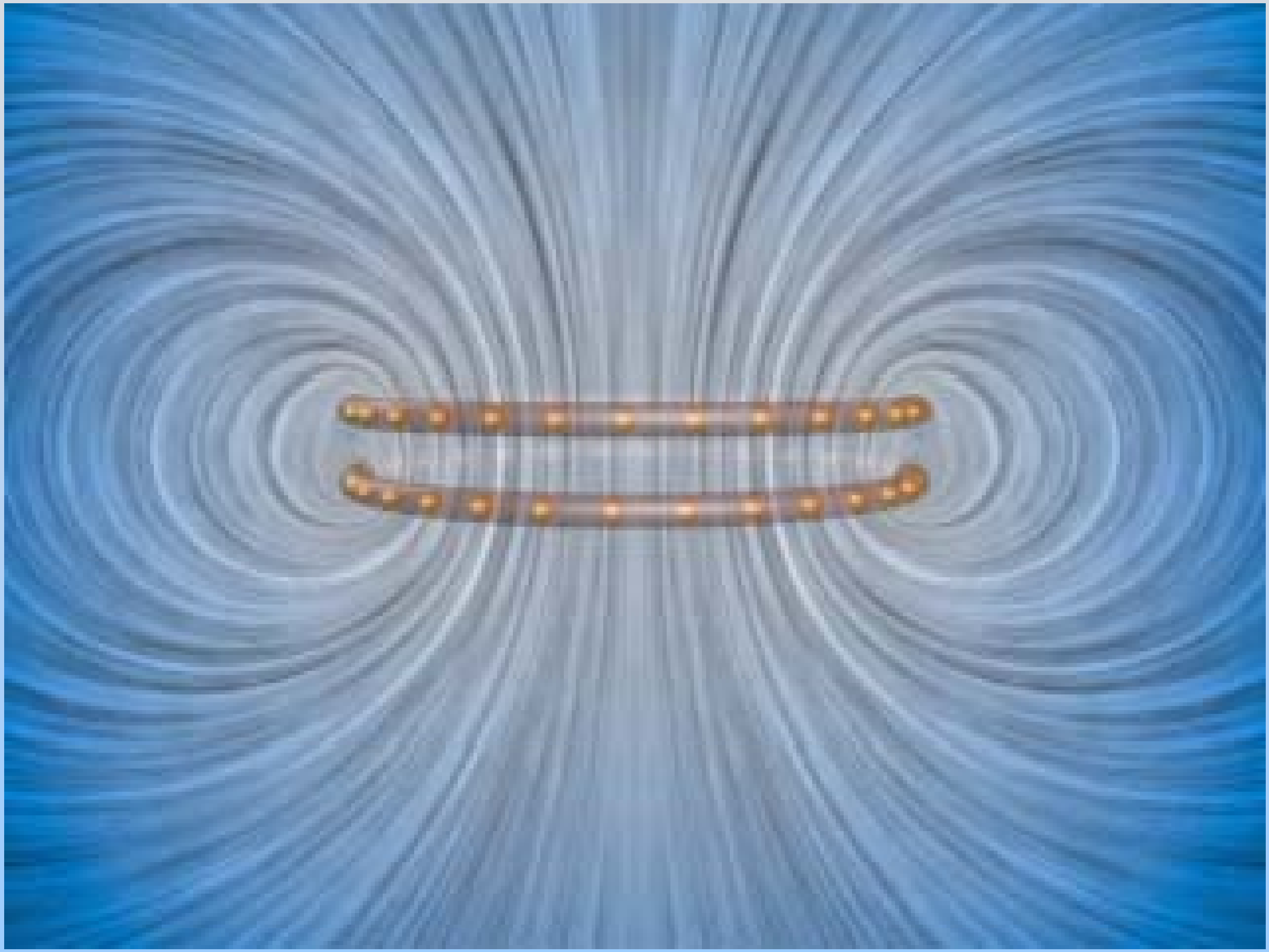
1. parallel currents that attract
2. parallel currents that repel
3. opposite currents that attract
4. opposite currents that repel

Force on Dipole from Dipole: Anti-Parallel Alignment



Link to animation

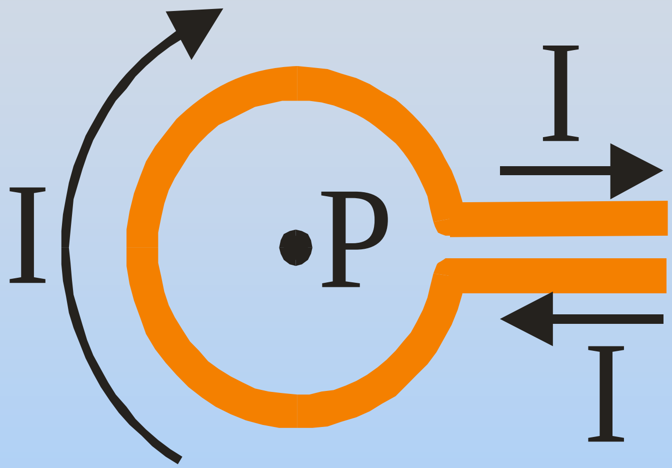
Force on Dipole from Dipole: Parallel Alignment



Link to animation

Example : Coil of Radius *R*

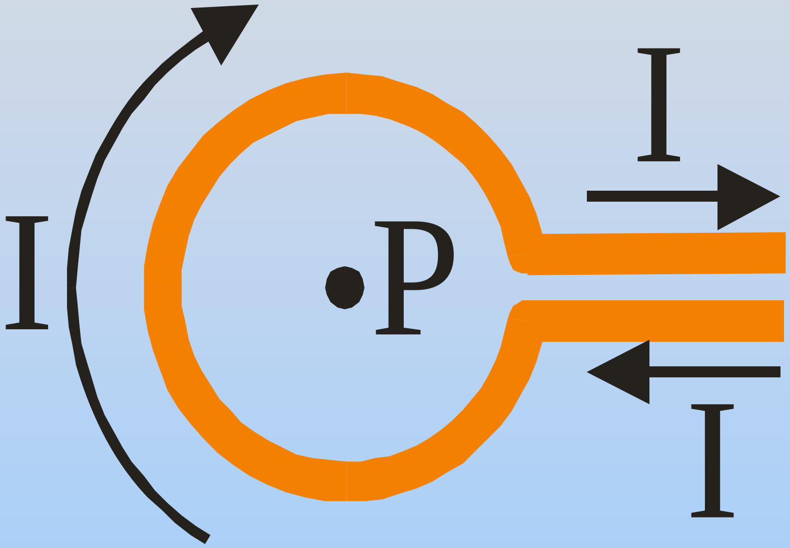
Consider a coil with radius *R* and current *I*



Find the magnetic field B at the center (P)

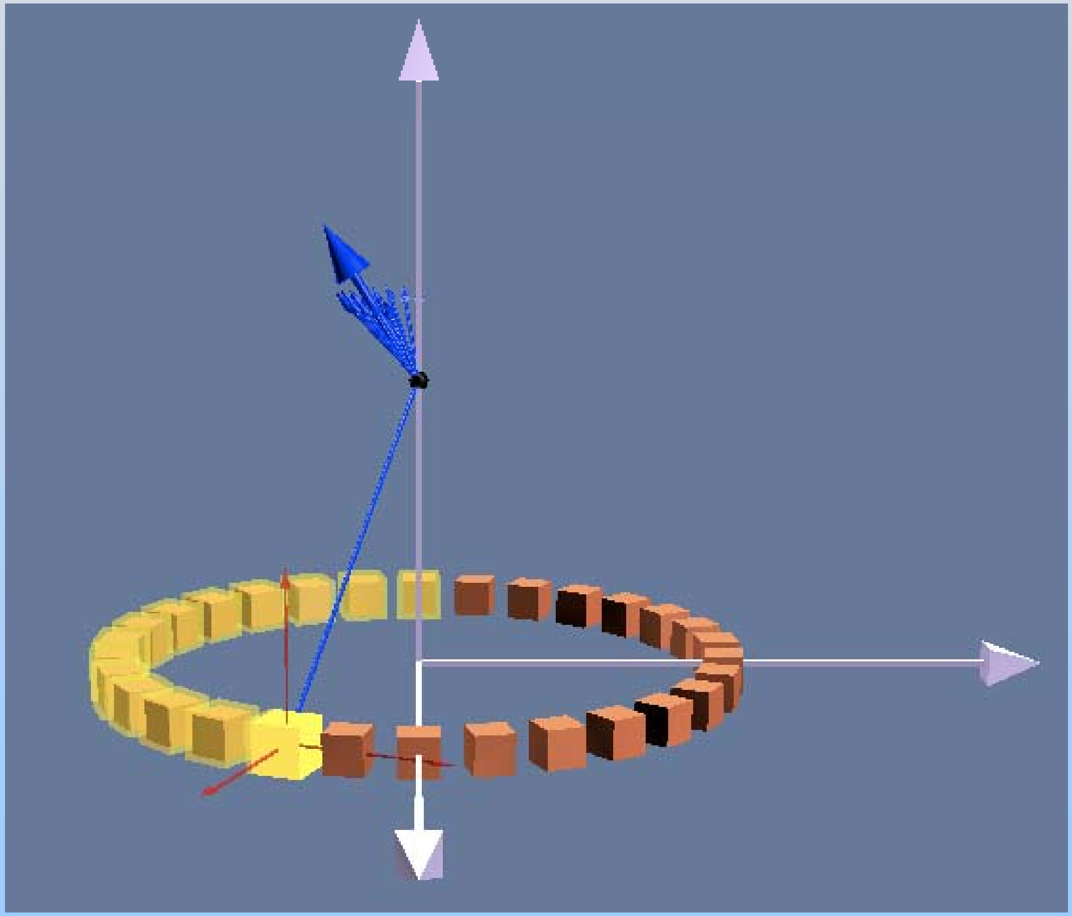
Example : Coil of Radius *R*

Consider a coil with radius *R* and current *I*



1) Think about it:
   - Legs contribute nothing
     I parallel to *r*
   - Ring makes field into page
2) Choose a ds
3) Pick your coordinates
4) Write Biot-Savart

Animation: Magnetic Field Generated by a Current Loop

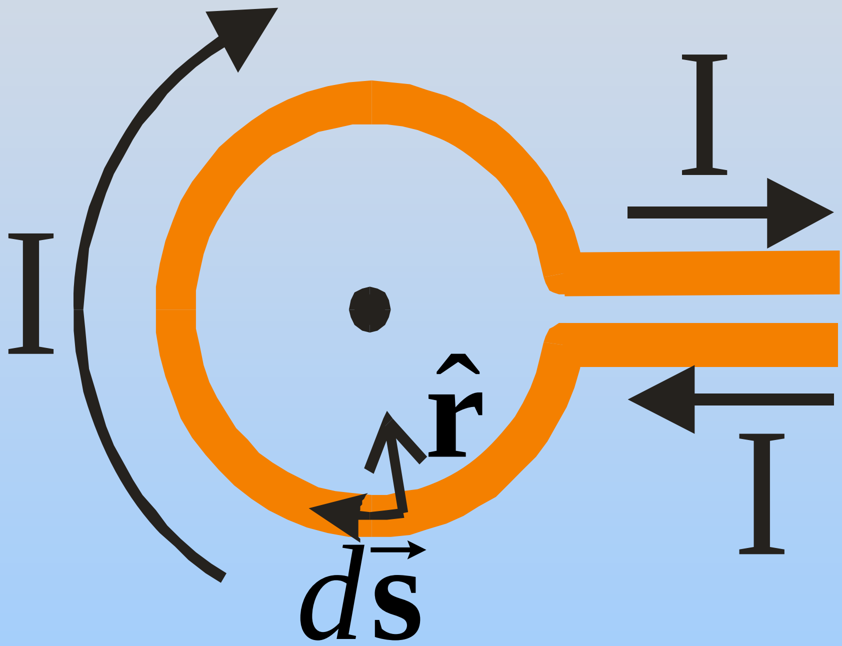


Link to Shockwave

Example : Coil of Radius *R*

In the circular part of the coil…

$$d\,\vec{\mathbf{s}} \perp \hat{\mathbf{r}} \quad \rightarrow \quad |\,d\,\vec{\mathbf{s}} \times \hat{\mathbf{r}}\,| = ds$$



Biot Savart:

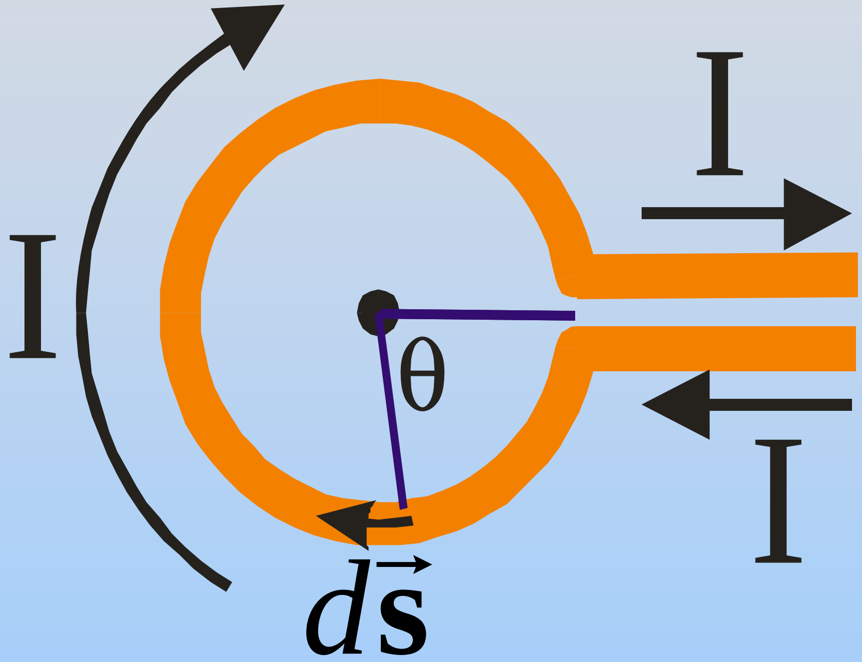
$$dB = \frac{\mu_0 I}{4\pi} \frac{\left| d\vec{\mathbf{s}} \times \hat{\mathbf{r}} \right|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{R\,d\theta}{R^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{d\theta}{R}$$

Example : Coil of Radius *R*

Consider a coil with radius *R* and current *I*



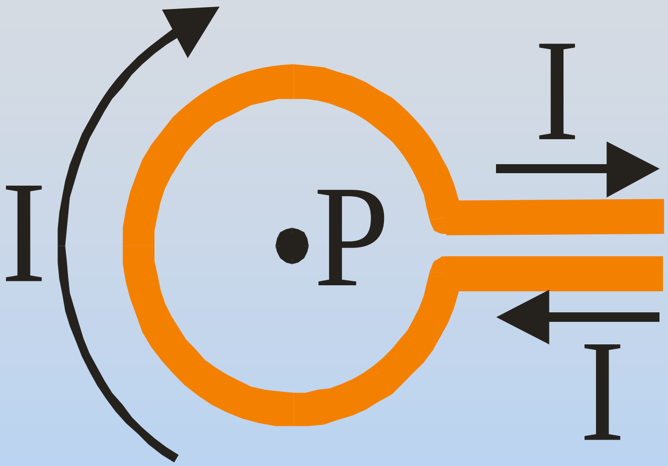
$$dB = \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

$$B = \int dB = \int_0^{2\pi} \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

$$= \frac{\mu_0 I}{4\pi R}\int_0^{2\pi} d\theta = \frac{\mu_0 I}{4\pi R}(2\pi)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R} \text{into page}$$

Example : Coil of Radius *R*



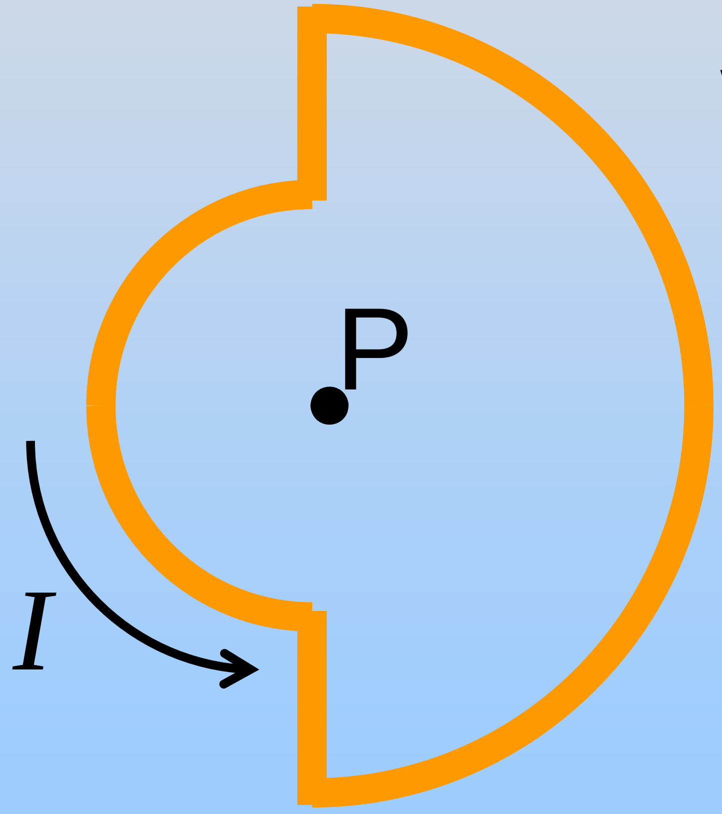
$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R} \text{ into page}$$

Notes:

- This is an EASY Biot-Savart problem:
  - No vectors involved
- This is what I would expect on exam

Problem:
B Field from Coil of Radius R

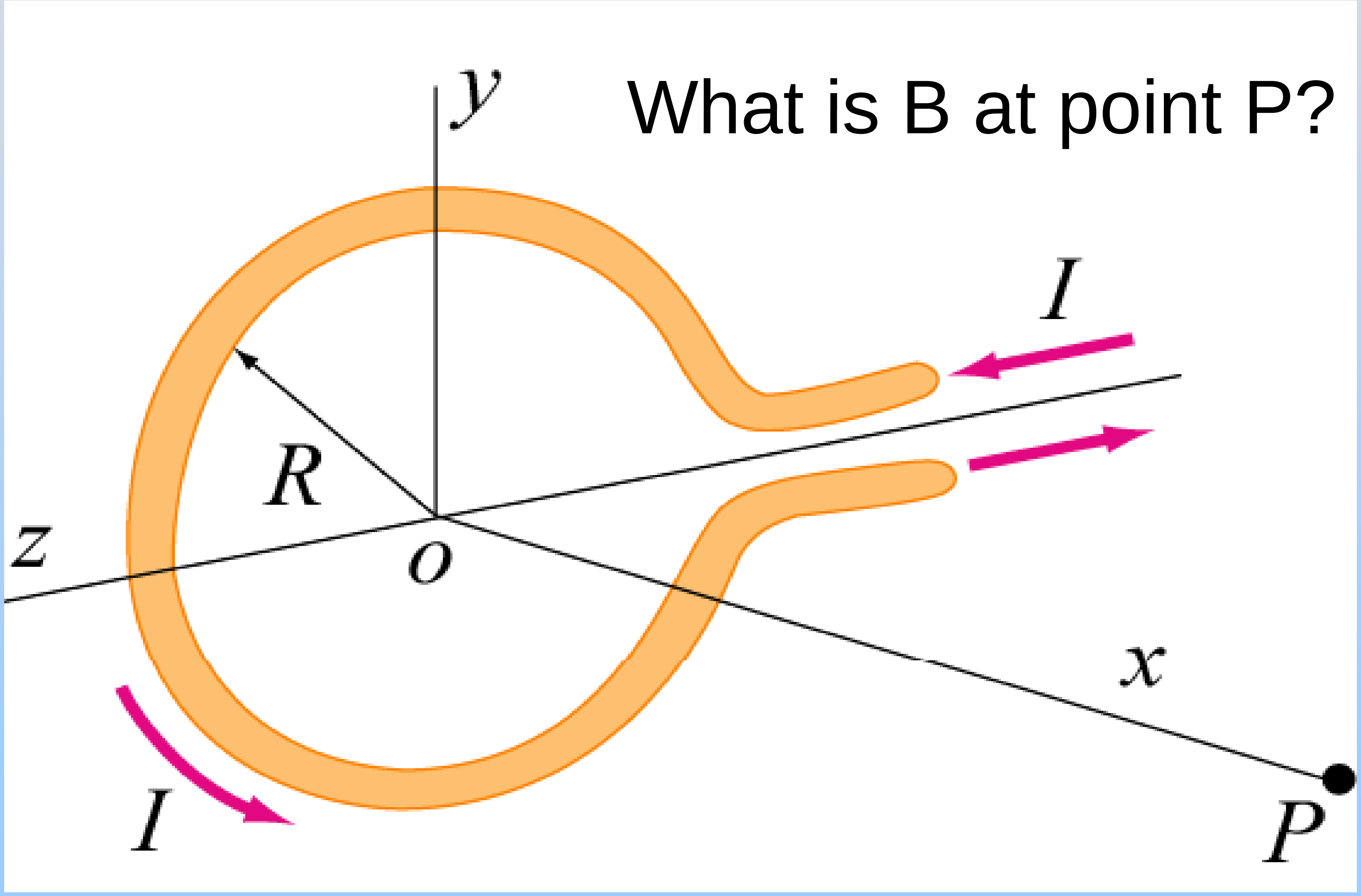
Consider a coil made of semi-circles of radii *R* and 2*R* and carrying a current I



What is **B** at point P?

Problem:
B Field from Coil of Radius R

Consider a coil with radius *R* and carrying a current *I*



WARNING:

This is much harder than the previous problem. Why??