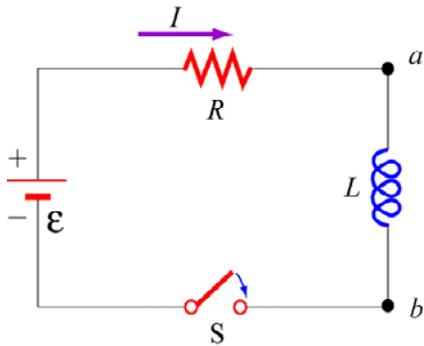


RL Circuits Challenge Problems

Problem 1:

RL Circuits



Consider the circuit at left, consisting of a battery (emf \mathcal{E}), an inductor L , resistor R and switch S .

For times $t < 0$ the switch is open and there is no current in the circuit. At $t = 0$ the switch is closed.

(a) Using Kirchhoff's loop rules (really Faraday's law now), write an equation relating the emf on the battery, the current in the circuit and the time derivative of the current in the circuit.

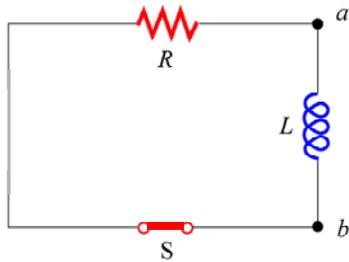
We know from thinking about it above that the results should look very similar to RC circuits. In other words:

$$I = A(X - \exp(-t/\tau))$$

- (b) Plug this expression into the differential equation you obtained in (a) in order to confirm that it indeed is a solution and to determine what the time constant τ and the constants A and X are. What would be a better label for A ? (HINT: You will also need to use the initial condition for current. What is $I(t=0)$?)
- (c) Now that you know the time dependence for the current I in the circuit you can also determine the voltage drop V_R across resistor and the EMF generated by the inductor. Do so, and confirm that your expressions match the plots in Fig. 2a or 2b.

Problem 2:

'Discharging' an Inductor



After a long time T the current will reach an equilibrium value and inductor will be “fully charged.” At this point we turn off the battery ($\mathcal{E}=0$), allowing the inductor to ‘discharge,’ as pictured at left. Repeat each of the steps a-c in problem 1, noting that instead of $\exp(-t/\tau)$, our expression for current will now contain $\exp(-(t-T)/\tau)$.

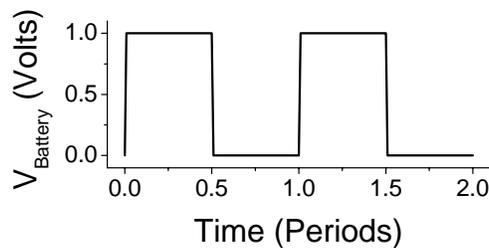
- (a) Faraday's law:
- (b) Confirm solution:
- (c) Determine V_R across resistor and the EMF generated by the inductor.

Problem 3:

A Real Inductor

Consider a coil that does not behave as an ideal inductor, but rather as an ideal inductor in series with a resistor. For this reason you have no way to independently measure the voltage drop across the resistor or the EMF induced by the inductor, but instead must measure them together. None-the-less, you want to get information about both. In this problem you will figure out how.

- (a) Hook up the circuit of problem 1 (with the ideal inductor L of that problem now replaced by a coil that is a non-ideal inductor – an inductor L and resistor r in series). The battery will periodically turn on and off, displaying a voltage as shown here:



Sketch the current through the battery as well as what a voltmeter hooked across the coil would show versus time for the two periods shown above. Assume that the period of the battery turning off and on is comparable to but longer than several time constants of the circuit.

- (b) How can you tell from your plot of the voltmeter across the coil that the coil is not an ideal inductor? Indicate the relevant feature clearly on the plot. Can you determine the resistance of the coil, r , from this feature?
- (c) In the lab you will find it easier to make measurements if you do NOT use an additional resistor R , but instead simply hook the battery directly to the coil. (Why? Because the time constant is difficult to measure with extra resistance in the circuit). Plot the current through the battery and the reading on a voltmeter across the coil for this case. We will only bother to measure the current. Why?
- (d) For this case (only a battery & coil) how will you determine the resistance of the coil, r ? How will you determine its inductance L ?

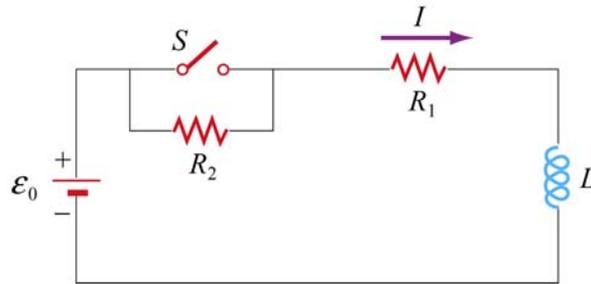
Problem 4:

The Coil

Consider a coil made of thin copper wire (**radius** ~ 0.25 mm) and has about 600 turns of average diameter 25 mm over a length of 25 mm. What approximately should the resistance and inductance of the coil be? The resistivity of copper at room temperature is around $20 \text{ n}\Omega\text{-m}$. Note that your calculations can only be approximate because this is not at all an ideal solenoid (where length \gg diameter).

Problem 5:

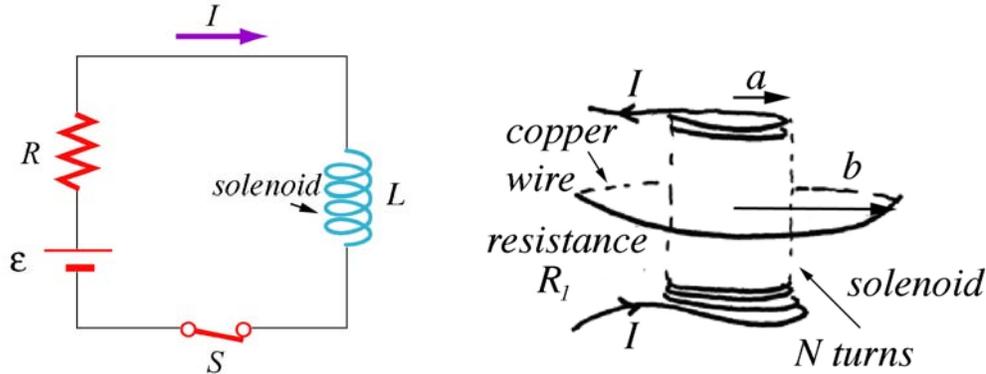
The LR circuit shown in the figure contains a resistor R_1 and an inductance L in series with a battery of emf ε_0 . The switch S is initially closed. At $t = 0$, the switch S is opened, so that an additional very large resistance R_2 (with $R_2 \gg R_1$) is now in series with the other elements.



- (a) If the switch has been *closed* for a long time before $t = 0$, what is the steady current I_0 in the circuit?
- (b) While this current I_0 is flowing, at time $t = 0$, the switch S is opened. Write the differential equation for $I(t)$ that describes the behavior of the circuit at times $t \geq 0$. Solve this equation (by integration) for $I(t)$ under the approximation that $\varepsilon_0 = 0$. (Assume that the battery emf is negligible compared to the total emf around the circuit for times just after the switch is opened.) Express your answer in terms of the initial current I_0 , and R_1 , R_2 , and L .
- (c) Using your results from (b), find the value of the total emf around the circuit (which from Faraday's law is $-LdI/dt$) just after the switch is opened. Is your assumption in (b) that ε_0 could be ignored for times just after the switch is opened OK?
- (d) What is the magnitude of the potential drop across the resistor R_2 at times $t > 0$, just after the switch is opened? Express your answers in terms of ε_0 , R_1 , and R_2 . How does the potential drop across R_2 just after $t = 0$ compare to the battery emf ε_0 , if $R_2 = 100R_1$?

Problem 6:

Consider the circuit shown in the figure, consisting of a battery (emf \mathcal{E}), a resistor with resistance R , a long solenoid of radius a , height H that has N turns and a switch S . Coaxial with the solenoid at the center of the solenoid is a circular copper ring of wire of radius b with $b > a$ and resistance R_1 . At $t = 0$ the switch S is closed.



- (a) What is the rate that the current is changing the instant the switch is closed at $t = 0$? Express your answer in terms of R , \mathcal{E} , and L , the self-inductance of the solenoid, as needed.
- (b) What is the self-inductance L of the solenoid? You may assume that the solenoid is very long and so can ignore edge effects. Express your answer in terms of μ_0 , a , b , H , N , R_1 , R , and \mathcal{E} as needed. **Answers without any work shown will receive no credit.**
- (c) What is the induced current in the copper ring at the instant the switch is closed at $t = 0$? Express your answer in terms of μ_0 , a , b , H , N , R_1 , R , and \mathcal{E} as needed.

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