

MATH 31A (Butler)
Practice for Midterm IIb

*Try to answer the following questions without the use of book, notes or calculator.
Time yourself and try to finish the questions in less than 50 minutes.*

1. (a) Use linearization to give an estimate for $\sqrt[3]{1017}$.
(b) Is the estimate given in part (a) too large or too small? Explain.
2. What is the area of the largest rectangle that you can make where the bottom edge is on the x -axis and the top two vertices lie on the parabola $y = 12 - x^2$?
3. A new toy is sweeping the nation, *The Silly Putty Ellipse*. It looks like any old ellipse but you can change the shape and it will always stay an ellipse with the same area (it comes straight out of the box as a circle with a radius of 6 inches)!
Let a and b be the length of the two axis of the ellipse. As you are playing with the ellipse you notice at one point when $a = 3$ inches that you are increasing the length of a by a rate of $\frac{1}{8}$ of an inch per second. At that moment how fast is b changing (include units)?
(Hint: the area of an ellipse with the two axis with length a and b is πab .)
4. Consider the following function which is continuous and differentiable for all x (you do not need to prove this!):

$$f(x) = \begin{cases} x^3 + 2x^2 - 4x + 2 & \text{if } x \leq 1; \\ 3x - 2 & \text{if } x > 1. \end{cases}$$

For the interval $-4 \leq x \leq 2$ find all values of c that satisfies the Mean Value Theorem.
(Hint: $f(-4) = -14$.)

5. The function $f(x) = \cos(x^3 - 2x)$ has a critical point at $x = 0$. Use the second derivative test to determine if it is a maximum or a minimum.