2 Signals and Systems: Part I

Recommended Problems

P2.1

Let \( x(t) = \cos(\omega_x(t + \tau_x) + \theta_x) \).

(a) Determine the frequency in hertz and the period of \( x(t) \) for each of the following three cases:

<table>
<thead>
<tr>
<th>( \omega_x )</th>
<th>( \tau_x )</th>
<th>( \theta_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ( \pi/3 )</td>
<td>0</td>
<td>2( \pi )</td>
</tr>
<tr>
<td>(ii) ( 3\pi/4 )</td>
<td>1/2</td>
<td>( \pi/4 )</td>
</tr>
<tr>
<td>(iii) ( 3/4 )</td>
<td>1/2</td>
<td>1/4</td>
</tr>
</tbody>
</table>

(b) With \( x(t) = \cos(\omega_x(t + \tau_x) + \theta_x) \) and \( y(t) = \sin(\omega_y(t + \tau_y) + \theta_y) \), determine for which of the following combinations \( x(t) \) and \( y(t) \) are identically equal for all \( t \).

<table>
<thead>
<tr>
<th>( \omega_x )</th>
<th>( \tau_x )</th>
<th>( \theta_x )</th>
<th>( \omega_y )</th>
<th>( \tau_y )</th>
<th>( \theta_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ( \pi/3 )</td>
<td>0</td>
<td>2( \pi )</td>
<td>( \pi/3 )</td>
<td>1</td>
<td>( -\pi/3 )</td>
</tr>
<tr>
<td>(ii) ( 3\pi/4 )</td>
<td>1/2</td>
<td>( \pi/4 )</td>
<td>( 11\pi/4 )</td>
<td>1</td>
<td>( 3\pi/8 )</td>
</tr>
<tr>
<td>(iii) ( 3/4 )</td>
<td>1/2</td>
<td>1/4</td>
<td>( 3/4 )</td>
<td>1</td>
<td>( 3/8 )</td>
</tr>
</tbody>
</table>

P2.2

Let \( x[n] = \cos(\Omega_x(n + P_x) + \theta_x) \).

(a) Determine the period of \( x[n] \) for each of the following three cases:

<table>
<thead>
<tr>
<th>( \Omega_x )</th>
<th>( P_x )</th>
<th>( \theta_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ( \pi/3 )</td>
<td>0</td>
<td>2( \pi )</td>
</tr>
<tr>
<td>(ii) ( 3\pi/4 )</td>
<td>2</td>
<td>( \pi/4 )</td>
</tr>
<tr>
<td>(iii) ( 3/4 )</td>
<td>1</td>
<td>1/4</td>
</tr>
</tbody>
</table>

(b) With \( x[n] = \cos(\Omega_x(n + P_x) + \theta_x) \) and \( y[n] = \cos(\Omega_y(n + P_y) + \theta_y) \), determine for which of the following combinations \( x[n] \) and \( y[n] \) are identically equal for all \( n \).

<table>
<thead>
<tr>
<th>( \Omega_x )</th>
<th>( P_x )</th>
<th>( \theta_x )</th>
<th>( \Omega_y )</th>
<th>( P_y )</th>
<th>( \theta_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ( \pi/3 )</td>
<td>0</td>
<td>2( \pi )</td>
<td>( 8\pi/3 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(ii) ( 3\pi/4 )</td>
<td>2</td>
<td>( \pi/4 )</td>
<td>( 3\pi/4 )</td>
<td>1</td>
<td>( -\pi )</td>
</tr>
<tr>
<td>(iii) ( 3/4 )</td>
<td>1</td>
<td>1/4</td>
<td>( 3/4 )</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

P2.3

(a) A discrete-time signal \( x[n] \) is shown in Figure P2.3.
Sketch and carefully label each of the following signals:

(i) \( x[n - 2] \)
(ii) \( x[4 - n] \)
(iii) \( x[2n] \)

(b) What difficulty arises when we try to define a signal as \( x[n/2] \)?

For each of the following signals, determine whether it is even, odd, or neither.

(a) \( x(t) \)
(b) \( x(t) \)

(c) \( x(t) \)
(d) \( x[n] \)
Consider the signal $y[n]$ in Figure P2.5.

(a) Find the signal $x[n]$ such that $E_0[x[n]] = y[n]$ for $n \geq 0$, and $O_0[x[n]] = y[n]$ for $n < 0$.

(b) Suppose that $E_0[w[n]] = y[n]$ for all $n$. Also assume that $w[n] = 0$ for $n < 0$. Find $w[n]$.

(a) Sketch $x[n] = \alpha^n$ for a typical $\alpha$ in the range $-1 < \alpha < 0$.

(b) Assume that $\alpha = -e^{-1}$ and define $y(t)$ as $y(t) = e^{\alpha t}$. Find a complex number $\beta$ such that $y(t)$, when evaluated at $t$ equal to an integer $n$, is described by $(-e^{-1})^n$.

(c) For $y(t)$ found in part (b), find an expression for $Re\{y(t)\}$ and $Im\{y(t)\}$. Plot $Re\{y(t)\}$ and $Im\{y(t)\}$ for $t$ equal to an integer.

Let $x(t) = \sqrt{2}(1 + j)e^{j \pi /4}e^{(-1+j)2\pi t}$. Sketch and label the following:

(a) $Re\{x(t)\}$

(b) $Im\{x(t)\}$

(c) $x(t + 2) + x^*(t + 2)$
P2.8

Evaluate the following sums:

(a) \( \sum_{n=0}^{5} 2 \left( \frac{3}{4} \right)^n \)

(b) \( \sum_{n=-2}^{6} b^n \)

(c) \( \sum_{n=-\infty}^{\infty} \left( \frac{2}{3} \right)^{2n} \)

Hint: Convert each sum to the form

\[ C \sum_{n=0}^{N-1} \alpha^n = S_N \quad \text{or} \quad C \sum_{n=0}^{\infty} \alpha^n = S_\infty \]

and use the formulas

\[ S_N = C \left( \frac{1 - \alpha^N}{1 - \alpha} \right), \quad S_\infty = \frac{C}{1 - \alpha} \quad \text{for } |\alpha| < 1 \]

P2.9

(a) Let \( x(t) \) and \( y(t) \) be periodic signals with fundamental periods \( T_1 \) and \( T_2 \), respectively. Under what conditions is the sum \( x(t) + y(t) \) periodic, and what is the fundamental period of this signal if it is periodic?

(b) Let \( x[n] \) and \( y[n] \) be periodic signals with fundamental periods \( N_1 \) and \( N_2 \), respectively. Under what conditions is the sum \( x[n] + y[n] \) periodic, and what is the fundamental period of this signal if it is periodic?

(c) Consider the signals

\[ x(t) = \cos \frac{2\pi t}{3} + 2 \sin \frac{16\pi t}{3}, \quad y(t) = \sin \pi t \]

Show that \( z(t) = x(t)y(t) \) is periodic, and write \( z(t) \) as a linear combination of harmonically related complex exponentials. That is, find a number \( T \) and complex numbers \( c_k \) such that

\[ z(t) = \sum_k c_k e^{j(2k\pi/T)t} \]

P2.10

In this problem we explore several of the properties of even and odd signals.

(a) Show that if \( x[n] \) is an odd signal, then

\[ \sum_{n=-\infty}^{\infty} x[n] = 0 \]

(b) Show that if \( x_1[n] \) is an odd signal and \( x_2[n] \) is an even signal, then \( x_1[n]x_2[n] \) is an odd signal.
(c) Let \( x[n] \) be an arbitrary signal with even and odd parts denoted by
\[
x_e[n] = Ev[x[n]], \quad x_o[n] = Od[x[n]]
\]
Show that
\[
\sum_{n = -\infty}^{+\infty} x[n] = \sum_{n = -\infty}^{+\infty} x_e[n] + \sum_{n = -\infty}^{+\infty} x_o[n]
\]

(d) Although parts (a)–(c) have been stated in terms of discrete-time signals, the analogous properties are also valid in continuous time. To demonstrate this, show that
\[
\int_{-\infty}^{+\infty} x^2(t) \, dt = \int_{-\infty}^{+\infty} x_e^2(t) \, dt + \int_{-\infty}^{+\infty} x_o^2(t) \, dt,
\]
where \( x_e(t) \) and \( x_o(t) \) are, respectively, the even and odd parts of \( x(t) \).

P2.11

Let \( x(t) \) be the continuous-time complex exponential signal \( x(t) = e^{j\omega_0 t} \) with fundamental frequency \( \omega_0 \) and fundamental period \( T_0 = 2\pi/\omega_0 \). Consider the discrete-time signal obtained by taking equally spaced samples of \( x(t) \). That is, \( x[n] = x(nT) = e^{j\omega_0 nT} \).

(a) Show that \( x[n] \) is periodic if and only if \( T/T_0 \) is a rational number, that is, if and only if some multiple of the sampling interval exactly equals a multiple of the period \( x(t) \).

(b) Suppose that \( x[n] \) is periodic, that is, that
\[
\frac{T}{T_0} = \frac{p}{q}, \quad (P2.11-1)
\]
where \( p \) and \( q \) are integers. What are the fundamental period and fundamental frequency of \( x[n] \)? Express the fundamental frequency as a fraction of \( \omega_0 T \).

(c) Again assuming that \( T/T_0 \) satisfies eq. \( (P2.11-1) \), determine precisely how many periods of \( x(t) \) are needed to obtain the samples that form a single period of \( x[n] \).