16 Sampling

Recommended Problems

P16.1

The sequence $x[n] = (-1)^n$ is obtained by sampling the continuous-time sinusoidal signal $x(t) = \cos \omega_0 t$ at 1-ms intervals, i.e.,

$$\cos(\omega_0 n T) = (-1)^n, \quad T = 10^{-3} \text{ s}$$

Determine three distinct possible values of $\omega_0$.

P16.2

Consider the system in Figure P16.2.

(a) Sketch $X_p(\omega)$ for $-9\pi \leq \omega \leq 9\pi$ for the following values of $\omega_0$.

(i) $\omega_0 = \pi$
(ii) $\omega_0 = 2\pi$
(iii) $\omega_0 = 3\pi$
(iv) $\omega_0 = 5\pi$

(b) For which of the preceding values of $\omega_0$ is $x_p(t)$ identical?

P16.3

In the system in Figure P16.3, $x(t)$ is sampled with a periodic impulse train, and a reconstructed signal $x_r(t)$ is obtained from the samples by lowpass filtering.

(a) Sketch $X_p(\omega)$ for $-9\pi \leq \omega \leq 9\pi$ for the following values of $\omega_0$.

(i) $\omega_0 = \pi$
(ii) $\omega_0 = 2\pi$
(iii) $\omega_0 = 3\pi$
(iv) $\omega_0 = 5\pi$

(b) For which of the preceding values of $\omega_0$ is $x_p(t)$ identical?
The sampling period $T$ is 1 ms, and $x(t)$ is a sinusoidal signal of the form $x(t) = \cos(2\pi f_0 t + \theta)$. For each of the following choices of $f_0$ and $\theta$, determine $x_s(t)$.

(a) $f_0 = 250$ Hz, $\theta = \pi/4$
(b) $f_0 = 750$ Hz, $\theta = \pi/2$
(c) $f_0 = 500$ Hz, $\theta = \pi/2$

P16.4

Figure P16.4 gives a system in which the sampling signal is an impulse train with alternating sign. The Fourier transform of the input signal is as indicated in the figure.

(a) For $\Delta < \pi/2\omega_M$, sketch the Fourier transform of $x_p(t)$ and $y(t)$.
(b) For $\Delta < \pi/2\omega_M$, determine a system that will recover $x(t)$ from $x_p(t)$.
(c) For $\Delta < \pi/2\omega_M$, determine a system that will recover $x(t)$ from $y(t)$.
(d) What is the maximum value of $\Delta$ in relation to $\omega_M$ for which $x(t)$ can be recovered from either $x_p(t)$ or $y(t)$.

Figure P16.4
Consider the system in Figure P16.5-1.

Figures P16.5-2 and P16.5-3 contain several Fourier transforms of \( x(t) \) and \( x_r(t) \). For each input spectrum \( X(\omega) \) in Figure P16.5-2, identify the correct output spectrum \( X_r(\omega) \) from Figure P16.5-3.

![Figure P16.5-1](image)

![Figure P16.5-2](image)
Suppose we sample a sinusoidal signal and then process the resultant impulse train, as shown in Figure P16.6-1.

\[ p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \]
The result of our processing is a value $Q$. As $\omega$ changes, $Q$ may change. Determine which of the plots in Figures P16.6-2 and P16.6-3 are possible candidates for the variation of $Q$ as a function of $\omega$.

**Figure P16.6-2**

![Graph showing $Q(\omega)$ vs. $\omega$ with a smooth curve]

**Figure P16.6-3**

![Graph showing $Q(\omega)$ vs. $\omega$ with a periodic waveform]

**Optional Problems**

**P16.7**

The sampling theorem as we have derived it states that a signal $x(t)$ must be sampled at a rate greater than its bandwidth (or, equivalently, a rate greater than twice its highest frequency). This implies that if $x(t)$ has a spectrum as indicated in Figure P16.7-1, then $x(t)$ must be sampled at a rate greater than $2\omega_s$. Since the signal has most of its energy concentrated in a narrow band, it seems reasonable to expect that a sampling rate lower than twice the highest frequency could be used. A signal whose energy is concentrated in a frequency band is often referred to as a bandpass signal. There are a variety of techniques for sampling such signals, and these techniques are generally referred to as bandpass sampling.
To examine the possibility of sampling a bandpass signal at a rate less than the total bandwidth, consider the system shown in Figure P16.7-2. Assuming that $\omega_1 > (\omega_2 - \omega_1)$, find the maximum value of $T$ and the values of the constants $A$, $\omega_a$, and $\omega_b$ such that $x_\ell(t) = x(t)$.

In Problem P16.7 we considered one procedure for bandpass sampling and reconstruction. Another procedure when $x(t)$ is real consists of using complex modulation followed by sampling. The sampling system is shown in Figure P16.8-1.
With \( x(t) \) real and with \( X(\omega) \) nonzero only for \( \omega_1 < |\omega| < \omega_2 \), the modulating frequency \( \omega_0 \) is chosen as \( \omega_0 = \frac{1}{2}(\omega_1 + \omega_2) \), and the lowpass filter \( H_1(\omega) \) has cutoff frequency \( \frac{1}{2}(\omega_2 - \omega_1) \).

(a) For \( X(\omega) \) as shown in Figure P16.8-2, sketch \( X_p(\omega) \).

(b) Determine the maximum sampling period \( T \) such that \( x(t) \) is recoverable from \( x_p(t) \).

(c) Determine a system to recover \( x(t) \) from \( x_p(t) \).

P16.9

Given the system in Figure P16.9-1 and the Fourier transforms in Figure P16.9-2, determine \( A \) and find the maximum value of \( T \) in terms of \( W \) such that \( y(t) = x(t) \) if \( s(t) \) is the impulse train

\[
s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)
\]
P16.10

Consider the system in Figure P16.10-1.

\[ p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \]

Figure P16.10-1

Given the Fourier transform of \( x_r(t) \) in Figure P16.10-2, sketch the Fourier transform of two different signals \( x(t) \) that could have generated \( x_r(t) \).

Figure P16.10-2

P16.11

Consider the system in Figure P16.11.
(a) If \(X(\omega) = 0\) for \(|\omega| > W\), find the maximum value of \(T, W_c,\) and \(A\) such that \(x_r(t) = x(t)\).

(b) Let \(X_1(\omega) = 0\) for \(|\omega| > 2W\) and \(X_2(\omega) = 0\) for \(|\omega| > W\). Repeat part (a) for the following.

(i) \(x(t) = x_1(t) * x_2(t)\)
(ii) \(x(t) = x_1(t) + x_2(t)\)
(iii) \(x(t) = x_1(t)x_2(t)\)
(iv) \(x(t) = x_1(10t)\)