1 Introduction

Recommended Problems

P1.1

Evaluate each of the following expressions for the complex number \( z = \frac{1}{2} e^{j\pi/4} \).

(a) \( Re(z) \)
(b) \( Im(z) \)
(c) \( |z| \)
(d) \( <z \)
(e) \( z^* \) (* denotes complex conjugation)
(f) \( z + z^* \)

P1.2

Let \( z \) be an arbitrary complex number.

(a) Show that
\[
Re(z) = \frac{z + z^*}{2}
\]
(b) Show that
\[
jIm(z) = \frac{z - z^*}{2}
\]

P1.3

Using Euler's formula, \( e^{j\theta} = \cos \theta + j \sin \theta \), derive the following relations:

(a) \( \cos \theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \)
(b) \( \sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \)

P1.4

(a) Let \( z = re^{j\theta} \). Express in polar form (i.e., determine the magnitude and angle for) the following functions of \( z \):

(i) \( z^* \)
(ii) \( z^2 \)
(iii) \( jz \)
(iv) \( zz^* \)
(v) \( \frac{z}{z^*} \)
(vi) \( \frac{1}{z} \)
(b) Plot in the complex plane the vectors corresponding to your answers to Problem P1.4a(i)–(vi) for \( r = \frac{3}{2}, \theta = \pi/6 \).

**P1.5**

Show that

\[
(1 - e^{j\alpha}) = 2 \sin \left( \frac{\alpha}{2} \right) e^{j(\alpha - \pi)/2}
\]

**P1.6**

For \( x(t) \) indicated in Figure P1.6, sketch the following:

(a) \( x(-t) \)
(b) \( x(t + 2) \)
(c) \( x(2t + 2) \)
(d) \( x(1 - 3t) \)

![Figure P1.6](image)

**P1.7**

Evaluate the following definite integrals:

(a) \( \int_{0}^{a} e^{-2t} \, dt \)
(b) \( \int_{2}^{\infty} e^{-3t} \, dt \)